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# **Fitted Poles/Residues: File Format, Transformations, Limitations**

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# Why Do We Need Approximation of Sampled Frequency-Domain Data with Rational Functions?

- It allows efficient simulation in time domain by recursive convolution, as opposed to direct convolution.
- Direct convolution:  $y(t_n) = \int_0^{t_n} x(\tau)g(t_n - \tau)d\tau$       $y(t_{n+1}) = \int_0^{t_{n+1}} x(\tau)g(t_{n+1} - \tau)d\tau$
- Fast overlap-save convolution works only for unidirectional systems, but not for S/Y/Z-parameters.
- Recursive convolution is a simple way to find  $y(t_{n+1})$  from known  $y(t_n)$ . If the impulse response of a linear system is exponential,  $g(t) = Ae^{-kt}$ , then

$$y(t_{n+1}) = G(h_{n+1})x(t_{n+1}) - z(t_{n+1}), \text{ where } z(t_{n+1}) = e^{-kh_{n+1}} z(t_n) + D(h_n, h_{n+1})x_n$$

- The step/impulse response is a sum of exponential functions if the transfer function could be written as partial fraction expansion, or in pole/residue form:

$$H(s) = H_\infty + \sum_{m=1}^M \frac{r_m}{s - p_m}$$

with  $M$  poles  $p_m$ ,  $m = 1 \dots M$ .

# Pole-Residue Representation

- The partial fractions could be converted into the ratio of two polynomials in  $s = i\omega$  however, the ***sum of rational fractions is more convenient*** and could be used directly for the purpose of time and frequency domain analysis.
- The model must be stable as it defines a linear non-autonomous system. ***Stable poles*** must be located on the left side of imaginary axis. This means that poles  $p_m$  ***have negative real part***.
- We do ***not allow poles with multiplicity larger than 1***, such as in the term  $\frac{1}{(s - p_m)^\mu}$ . The reason is not our inability to represent them in pole/residue form, but the fact that time domain response from the systems with multiple poles doesn't allow recursive convolution. In practice, multiple poles could often be replaced by a set of close but distinct poles.
- For completeness, we can add reactive linear asymptotic term, which may be present in Y/Z parameters:

$$H(s) = H_\infty + \sum_{m=1}^M \frac{r_m}{s - p_m} + sQ$$

# Realness. Grouping Complex Conjugate Pairs Together

- In our practical work we expect that the time domain variables of electrical network (voltages, currents, scattered waves) are real functions of time. This assumes model “realness” which implies that all poles/residues appear in complex-conjugate pairs, and that the coefficients of the rational polynomials representing the dependence are real, too.
- It makes sense to “enforce” realness of the data by using the format which wouldn’t allow unmatching fractions with complex poles.
- Let’s take a complex conjugate pole/residue pair and convert it as follows:

$$Y_m = \frac{c_m}{s - p_m} + \frac{c'_m}{s - p'_m} = \frac{c_m / (-p_m)}{1 + s / (-p_m)} + \frac{c'_m / (-p'_m)}{1 + s / (-p'_m)} = \frac{1}{2} \left[ \frac{2c_m / (-p_m)}{1 + s / (-p_m)} + \frac{2c'_m / (-p'_m)}{1 + s / (-p'_m)} \right] \quad (1)$$

With denotations:

$$\Omega_m = \alpha_m + i\omega_m = -p_m, A_{1m} = \text{Re}\{c_m / (-p_m) + c'_m / (-p'_m)\}, A_{2m} = -\text{Im}\{c_m / (-p_m) - c'_m / (-p'_m)\}$$

(1) can be written as

$$Y_m = \frac{1}{2} \left[ \frac{A_{1m} - iA_{2m}}{1 + s / (\alpha_m + i\omega_m)} + \frac{A_{1m} + iA_{2m}}{1 + s / (\alpha_m - i\omega_m)} \right] \quad (2)$$

# Grouping Complex Conjugate Pairs Together

$$Y_m(s) = \frac{1}{2} \left[ \frac{A_{1m} - iA_{2m}}{1 + s / (\alpha_m + i\omega_m)} + \frac{A_{1m} + iA_{2m}}{1 + s / (\alpha_m - i\omega_m)} \right]$$

- This form has several advantages:
  - Guarantees that complex conjugate poles come in pairs
  - Requires 4 real numbers to define complex conjugate pair with  $[\alpha_m \ \omega_m \ A_{m1} \ A_{m2}]$
  - Turns into a single real pole if  $\omega_m=0$  and  $A_{2m}=0$
  - Pole stability is enforced by having  $\alpha_m > 0$
  - For convenience, we can assume  $\omega_m \geq 0$ , because the order of poles inside the pair is not fixed
  - Coefficients  $A_{m1}$ ,  $A_{m2}$  are magnitudes.  $A_{m1}$  defines contribution of the term  $Y_m$  into DC value
  - We can use the term  $Y_m$  to define the value  $D$  at infinite frequency by choosing the values e.g.,  $[10^{20} \ 0 \ D \ 0]$

# Multiport Models: Common Set of Poles for All Matrix Components or Individual Poles for Each?

- The choice is not simple, as it depends on different circumstances. If the object we simulate is a ***lumped circuit***, it allows state-space representation and can be defined by a common set of poles. If it has ***large electrical length*** (long cable, traces, etc.) it may not have accurate state-space representation and may require individual pole selection for each matrix component, possibly with delay.
- With common poles we have more choices in passivity enforcement methods, and some post-fit transformations, but with individual sets of poles – better chance to get more accurate fit.
- The pole/residual file format must allow both cases. If we have a separate table of poles/residues for each matrix component, the poles could still be the same in each table, or this fact could be used to reduce the size of the tables (somewhat similar to [Matrix Format] in the Touchstone 2.0 definition).

# Models with Delay

- We found it useful to represent some dependences as a product of the delay operator and the sum of pole/residue terms. For example, **insertion loss** dependence of the cable or long trace may contain several ns delay, while other components of the same model don't.
- Separating the delay  $H_T(s) = e^{-sT} H_0(s)$  serves two purposes. First, we **remove initial noise from the time domain solution**, and most importantly, the **remainder becomes a much simpler function to fit**, which improves overall fit accuracy.
- Delay operator by itself is a causal function and can be represented in pole/residue form. However, large delay may considerably increase complexity of the fit.
- Of course, the delay extraction must be conservative, to avoid non-causality. The delay could be estimated e.g., by analyzing the unwrapped phase, or finding time domain response by IFFT.
- During passivity evaluation, we'd need to add the delay back to such components. But prior to residue perturbation, the required correction function must be multiplied by the inverse delay operator, in order to obtain necessary correction to the delay-less remainder.

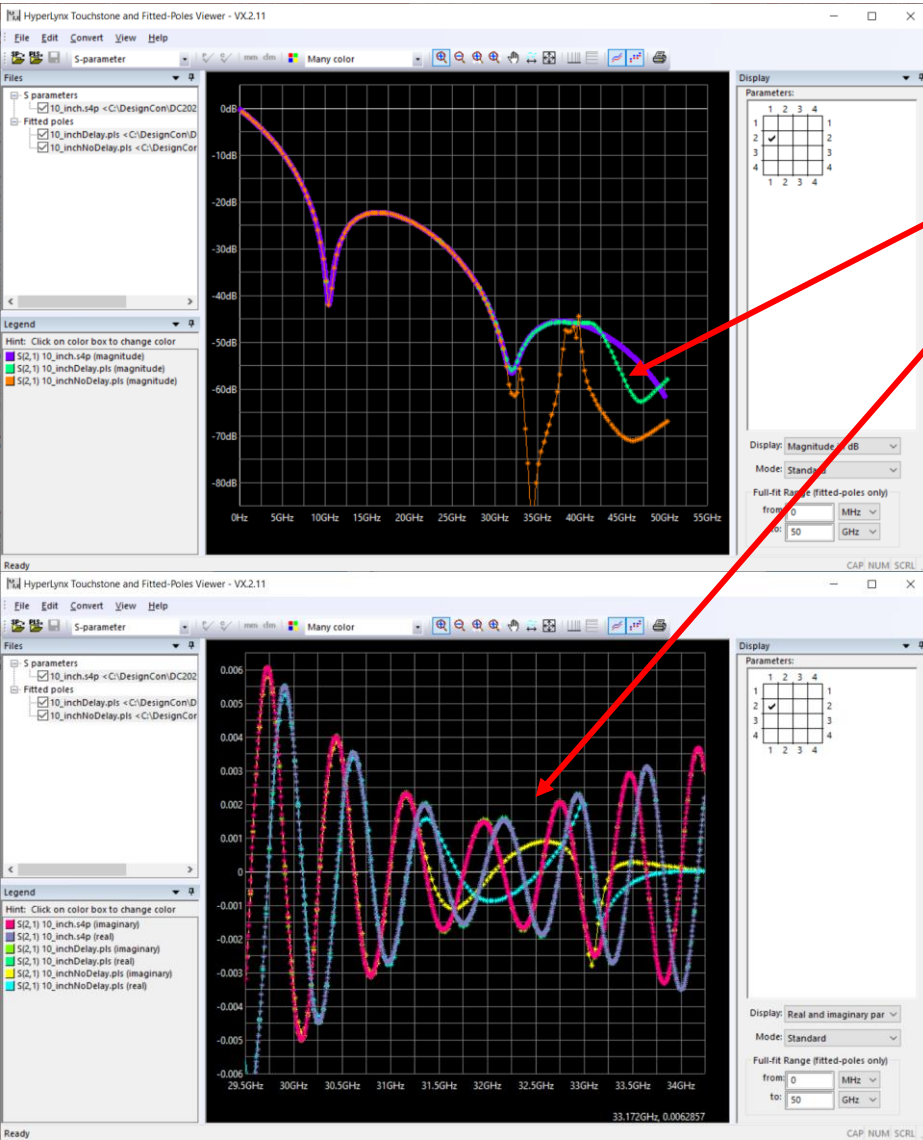
# Systems with Delay (10-inch diff pair)

By considering delay extraction we get

Better accuracy

Fewer poles

Smaller file size



10\_inch.pls - Notepad

```
File Edit Format View Help
3.86502623523495e+07 -0.00000000000000e+00 -8.41554565033922e-03 0.00000000000000e+00
1.00000000000000e+21 0.00000000000000e+00 2.10450816965376e-02 0.00000000000000e+00
98
1.16619129572801e+08 3.98809590017720e+10 -2.73024759920506e-05 5.02466246780508e-05
2.16372427295099e+08 3.9286845273238e+10 6.20534956308890e-05 1.16865586404727e-04
2.44592436898632e+08 3.8743813907550e+10 1.60223345362124e-04 -3.35600013420313e-05
2.03003578969692e+08 3.8211012746429e+10 -6.09208852941108e-06 -9.44458491823659e-05
9.19732385178175e+07 3.30715261847728e+10 2.02190656947422e-05 2.00998561413561e-06
3.90391553653668e+08 3.11439217778293e+10 -5.16620664052498e-04 -2.53880752461839e-04
6.52165932930182e+08 3.03793329599216e+10 -8.13728139061741e-03 1.67384840885711e-03
8.16586246794974e+08 2.98035147950983e+10 -1.19838927127943e-02 4.91060533770031e-02
9.18300310388615e+08 2.92472508748922e+10 1.42854874301229e-01 9.39393111259226e-02
9.70436678430132e+08 2.86892456457503e+10 3.02287776419399e-01 -2.06255658490392e-01
9.83026077553299e+08 2.81046405680894e+10 7.08307237512010e-03 -5.58480965317260e-01
```

10\_inchDelay.pls - Notepad

```
File Edit Format View Help
3.86502623523495e+07 -0.00000000000000e+00 -8.41554565033922e-03 0.00000000000000e+00
1.00000000000000e+21 0.00000000000000e+00 2.10450816965376e-02 0.00000000000000e+00
Delay: 1.263501e-09
43
1.84061251118162e+09 4.18307710480626e+10 -2.80429829387243e-04 1.47079855452111e-03
5.89456088110203e+09 3.67251018170311e+10 5.82336197411629e-03 1.82061738131804e-02
5.18968910327939e+09 3.23488777049619e+10 1.00811354769097e-01 7.90445863621127e-02
6.06546143218476e+09 2.84007680666912e+10 7.69872223535576e-01 -3.58734138117119e-01
3.7220888412863e+08 2.76746673591434e+10 -2.02740088894807e-06 3.00514481802362e-06
6.47691607074774e+09 2.46548388851417e+10 -1.36385747080121e+00 -3.02151858981163e+00
3.63975682269968e+08 2.49574331262561e+10 3.09919330189302e-06 -2.34122960765144e-06
6.53242543883198e+09 2.10096435428231e+10 -6.40851043961784e+00 3.90105899357689e+00
3.56315292420744e+08 2.23446954469707e+10 -3.76113980806339e-06 1.52673992928458e-06
6.23552998047704e+09 1.74155063405902e+10 5.96311647301732e+00 8.22375985876519e+00
```

Name	Date modified	Type	Size
10_inch.s4p	3/13/2019 ...	S4P File	7,392 KB
10_inchDelay.pls	3/29/2022 ...	PLS File	99 KB
10_inchNoDelay.pls	3/29/2022 ...	PLS File	145 KB



# Table of Poles/Residues or Equivalent Circuit?

- Once the fit is found, it can be converted into a linear equivalent circuit. There are several formats which differ by the choice of network components. Most frequently, they use R/L/C components with controlled voltage and/or current sources. They can also incorporate delays, allow different port normalization impedance (S-parameters) or specify the term  $sQ$  for Y and Z types.
- The advantage of equivalent circuits written e.g., in SPICE format, is portability as it doesn't use the table of the fitted pole/residues.
- However, there are serious drawbacks. First, accuracy. Capacitors / inductors create differential equations which circuit simulator solves numerically by finite difference methods (implicit Euler, Trapezoidal, etc.). Such methods create truncation error proportional to a certain power of the time step. In contrast, recursive convolution doesn't do this because it uses analytically found integral form of the solution.
- Another problem is speed: a simple fit of a 4-port S-parameters will produce many additional equations, which should be solved simultaneously on every Newton iteration at every time step.
- The advantage of tables representing poles/residues is fast update of state variables inside the model instance which doesn't add more equations than the number of model's terminals.
- ***The lack of the standard on pole/residue model definition format is what prevents us from using it across different tools***

# PLS File Format Currently Supported

- This format is used in Mentor / Siemens EDA for about 20 years. It is simple but proved sufficient for different problems/models.
- The file header defines:
  - parameter type (S, Y, Z...), # of ports, port normalization impedances
  - *Mixed mode order (similar to Touchstone 2.0 format) [optional]*
  - *Frequency and value units [optional, by default assume Hz, Ohm, 1/Ohm]*
- Unlike the touchstone file, the data must be ordered per component, not per frequency
- For each matrix component  $(k, n)$ ,  $k = 1...N$ ,  $n = 1...N$ , define table of poles/residues using 4 numbers  $[\alpha_m \ \omega_m \ A_{m1} \ A_{m2}]$  in each line,  $m = 1...M$ , where  $M$  is the number of real poles and complex conjugate pairs defined for the component  $(k, n)$ . The number of lines  $M$  must be defined prior to the table.
- If necessary, the extracted delay or the factor at linearly growing imaginary component could be defined.

# PLS File Format Currently Supported

## Header

- The **header** defines parameter type (S, Y, Z), and the number of model ports. Normalization impedances are specified on a separate line. Examples:

```
S 4
R0:  50  20  30  40

Y 4
R0:  50  50  50  50

Z 4
R0:  50  50  50  50
```

## Matrix component $(i, j)$ , $i=1...N$ , $j=1...N$

Each **matrix component** section has its own **header**. It defines extracted delay (if relevant), linear asymptote  $Q$  for  $sQ$  dependence (for Y or Z parameters, if exists). This header doesn't show the indices of the matrix component but assumes that they follow the designated order, like S11, S12, S13....

Examples:

**35** (a) only the number of poles/residues for this component

**delay: 1.26351e-09** (b) delay and the number of poles/residues  
**43**

**asympt: 0.83754e-12** (c) factor at complex frequency, for Y/Z types only  
**27**

The number of poles/residues includes a high frequency pole that defines the value at infinity. Each pole/residue component has the form  $[\alpha_m \ \omega_m \ A_{m1} \ A_{m2}]$  and must be defined on a separate line:

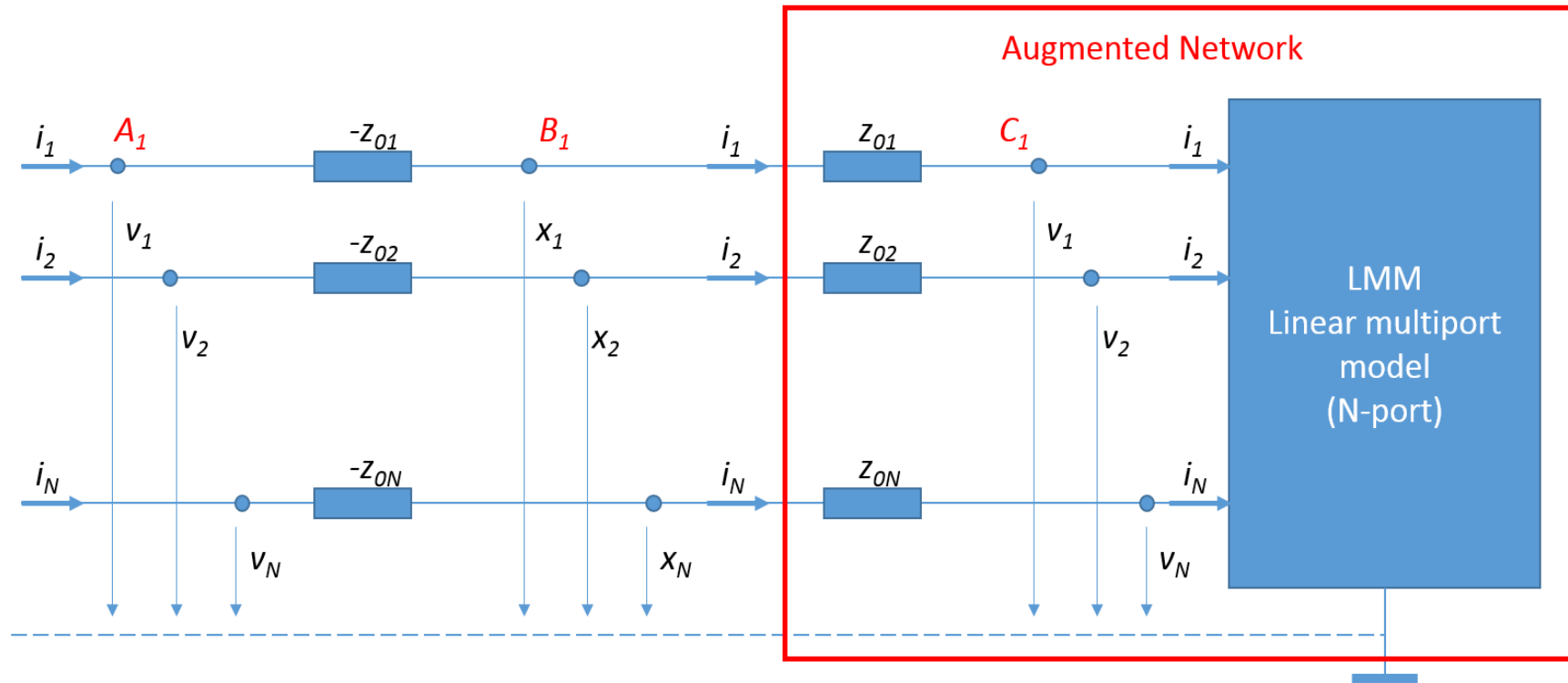
```
1.60981891306855e+08  6.03830005978569e+09  -2.15363238798792e-06  1.96534688582861e-05
2.93321810887676e+09  1.91770843721616e+09  -1.05426912887832e+01  -8.82630433918342e+00
1.23990373548953e+08  4.39994357143840e+09  1.25728612812034e-05  2.13669372820529e-05
1.57193681614524e+08  3.10437931944199e+09  5.10972708034658e-05  -1.15663907003945e-05
2.83363448768380e+07  2.10218276754607e+09  3.07314704488451e-06  5.16091015967049e-06
...
1e20  0.0  0.321123423421  0.0
```

*Note: Example files are available*

# PLS File Format, Possible Changes or Additions

- Any comments preceded by '!', allowed before the header line, such as port mapping syntax, etc. could be defined.
- [Mixed Mode Order] keyword and the data, per Touchstone 2.0 definition
- A cleaner way to define the value at infinite frequency (we use a real pole at very high frequency)
- Support for the case with all common poles. This will require the keyword, and allow the poles being listed after the header. Then, for each matrix components we can specify residues and the value at infinity. Perhaps, no support for delay in this case. Expected file size decrease by about factor 2
- If the model is reciprocal, this could be reflected by another keyword. The size of the matrix can be further reduced
- No need to use different units. We can agree that pole frequencies are defined in [Hz], and residues in natural units: Y – Siemens, Z – Ohms, S - dimensionless.

# Fitted S-parameters in Time Domain Simulation



Model stamp (TD)

Model stamp (FD)

AN Conductance Matrix

$$\begin{bmatrix} I_k \\ 0 \end{bmatrix} = \begin{bmatrix} -Z_0^{-1} & Z_0^{-1} \\ Z_0^{-1} & -Z_0^{-1} + J_{AN} \end{bmatrix} \begin{bmatrix} V_k \\ X_k \end{bmatrix} + \begin{bmatrix} 0 \\ j_{k-1} \end{bmatrix}$$

$$\begin{bmatrix} I \\ 0 \end{bmatrix} = \begin{bmatrix} -Z_0^{-1} & Z_0^{-1} \\ Z_0^{-1} & -Z_0^{-1} + Y_{AN} \end{bmatrix} \begin{bmatrix} V \\ X \end{bmatrix}$$

$$Y_{AN} = \frac{1}{2} Z_0^{-1/2} (E - S) Z_0^{-1/2}$$

Note: Matrix components with delay multipliers make variable contributions into  $j_{k-1}$  only

# Transformation of Models Represented by Poles-Residues

- Compared to the sampled frequency dependent model (Touchstone file), not all matrix transformations are easily available that preserve accuracy and passivity of the fitted model.
- The simplest transformations are (a) mode conversion (between STD and MM), multiplication on equalization filter represented by poles/residues, port reduction that assumes matched termination of unused ports (R0 for S-parameters, short circuit for Y, open circuit for Z)
- Model type conversion (between S-Y-Z) is more complicated, and so is port terminations on arbitrary load.
- The complexity of transformations depends on whether the model allows state-space representation or not.

# Mode Conversion and Multiplication

- Conversion from standard into mixed mode (and inverse):

$$s_{i,j,diff} = \frac{1}{2}(s_{ii} - s_{ij} - s_{ji} + s_{jj}) \quad s_{i,j,comm} = \frac{1}{2}(s_{ii} + s_{ij} + s_{ji} + s_{jj})$$

We only need to combine the factors at the same poles (common set of poles) or add more pole/residue components into fit representation

- Multiplication on a filter represented by poles/residues (e.g., channel equalizer, transition or noise filter, etc.) The product of the pole/residue pair with distinct poles can be replaced by their sum, with modified residues:

$$\frac{A_k}{s - p_k} \times \frac{A_m}{s - p_m} = \frac{B_k}{s - p_k} + \frac{B_m}{s - p_m}$$

The product of the two scalar dependencies with  $N1$  and  $N2$  pole/residue components will result in  $(N1+N2)$  such components. Each component is either a single real, or a pair of complex-conjugate poles/residues.

# Model Type conversion (between Y and Z)

- State-space system representation (SSR) and poles/residues. With common set of poles we can define the system of equations:

$$\begin{bmatrix} \dot{X}(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} X(t) \\ U(t) \end{bmatrix} \text{ or } \begin{bmatrix} 0 \\ Y(s) \end{bmatrix} = \begin{bmatrix} (A-sE) & B \\ C & D \end{bmatrix} \begin{bmatrix} X(s) \\ U(s) \end{bmatrix} \quad (1)$$

can be converted into a transfer function which gives poles/residues:

$$Y(s) = H(s)U(s) = \left[ -C(A-sE)^{-1}B + D \right] U(s) \quad (2)$$

- Conversion between Y and Z means swapping vectors Y and U. This gives:

$$\begin{bmatrix} 0 \\ U(s) \end{bmatrix} = \begin{bmatrix} (A_n - sE) & B_n \\ C_n & D_n \end{bmatrix} \begin{bmatrix} X_n(s) \\ Y(s) \end{bmatrix} \quad (3)$$

- If model type transformation exists, the new matrices could be found from (1):

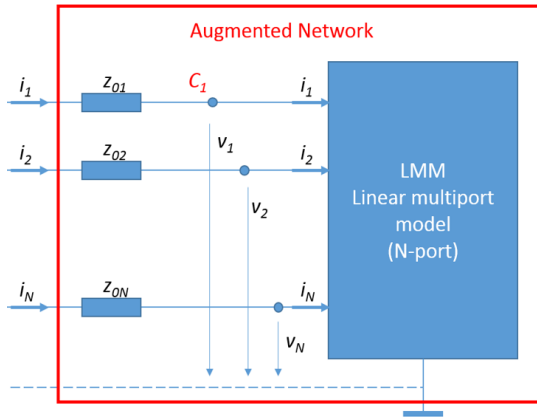
$$A_n = A - BD^{-1}C, B_n = BD^{-1}, C_n = -D^{-1}C, D_n = D^{-1} \quad (4)$$

Then, the poles/residues of the new model could be defined from (2) using updated matrices (4)



# Model Type conversion (between S and Y or Z)

- There exist direct conversions between S and either Y or Z parameters.
- Let's recall the augmented network:



$$Y_{AN} = \frac{1}{2} Z_0^{-1/2} (E - S) Z_0^{-1/2}$$

- Poles/residues of  $Y_{AN}$  could be found from the fitted S-parameters
- Inverse of  $Y_{AN}$  (as explained in previous slide) gives us Z-parameters plus the diagonal containing port normalizing impedances. Therefore Z-parameters can be easily found.
- Then, Y-parameters can be found from Z by another inversion. However, there is a direct way of finding Y-parameters from S-parameters.

# Transformation of Models Represented by Poles-Residues

- Transformations of models given in pole/residue form are possible, but more complicated than similar conversions for the sampled dependencies (Touchstone data)
- Termination of ports using arbitrary conductance/impedance defined by poles, or S-parameter matrix re-normalization is more difficult than transformations considered above.
- Transformations become even more complicated when matrix components are defined by different sets of poles. In this case, SSR requires combining all poles together which considerably increases the size of matrix  $A$  and the number of state variables.
- Such difficulties aggravate when working with multiport models. For example, we had to work with some S-parameter models with as many as 400-500 ports.
- At some point, a reasonable solution is to re-sample the fitted model, make necessary conversions/transformations and then re-fit the resulted matrix dependence

# Conclusion

- The pole/residue format is the most convenient data representation for fast time-domain simulation. It also works for frequency-domain simulations and model extraction
- This format must be standardized to allow the use of pole/residue tables across different simulation and analysis tools
- In this presentation we discussed one of several possible solutions
- The goal is to collect all proposals on this topic and choose a sufficiently universal and convenient pole/residue file format.