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Draft
Standard for Validation of Computational Electromagnetics Computer Modeling and Simulations

Prepared by the P1597.1 Working Group of the
IEEE Electromagnetic Compatibility Society (EMC-S) Standards Development Committee

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Draft Standard for Validation of Computational Electromagnetics Computer Modeling and Simulations

1. Overview

1.1 Scope

This standard defines a method to validate computational electromagnetics (CEM) computer modeling and simulation (M&S) techniques, codes, and models. It is applicable to a wide variety of electromagnetic (EM) applications including but not limited to the fields of electromagnetic compatibility (EMC), radar cross section (RCS), signal integrity (SI), and antennas. Validation of a particular solution data set can be achieved by comparison to the data set obtained by measurements, alternate codes, canonical, or analytic methods.

1.2 Purpose

This standard provides a formal mechanism for comparing the results of various CEM techniques, codes, and models in a repeatable way against a set of "golden" benchmarks, including standard validation and canonical problem sets. These data are based on theoretical formulations, or obtained as a result of performing high-quality measurements and, in certain cases, based on accurate analyses that have undergone and withstood peer validation.

1.3 Background

In general, CEM techniques and codes, and the manner in which they are used to analyze a given problem, can produce quite different results. These results are affected by the way in which the underlying physic formalisms have been implemented within the codes including the mathematical basis functions, numerical solution methods, numerical precision, and the use of building blocks (primitives) to generate computational models.

Despite all CEM codes having their basis in Maxwell's equations of one form or another, their accuracy and convergence rate depends on how the physics equations are cast (e.g., integral or differential form, frequency or time domain), what numerical solver approach is used (full or partial wave, banded or partitioned matrix, non-matrix), inherent modeling limitations, approximations, and so forth. The physics formalism, available modeling primitives (canonical surface or volumetric objects, wires, patches, facets), analysis frequency, and time or mesh discretization further conspire to affect accuracy, solution convergence, and overall validity of the computer model.

The critical areas that must be addressed include model accuracy, convergence, and techniques or code validity for a given set of canonical, standard validation, and benchmark models. For instance, uncertainties may arise when the predicted results using one type of CEM technique do not agree favorably or consistently with the results of other techniques or codes of comparable type or even against measured data on benchmark models. Furthermore, it can be difficult to compare the results between certain techniques or codes despite their common basis in Maxwell's equations. Exceptions can be cited, in

particular, when comparing the results of "similar" codes grouped according to their physics, solution methods, and modeling element domains. Nevertheless, disparities even among codes in a certain "class" have been observed. Many examples can be cited where fairly significant deviations have been observed between analytical or computational techniques and empirical based methods. Differences are not unexpected, but the degree of disparity in certain cases cannot be readily explained nor easily discounted, which has led to the often asked question, "*…which result is accurate?*" Annex B provides guidelines on how to apply the right CEM technique or code to the CEM M&S problem at hand.

2. Normative references

The following referenced documents are indispensable for the application of this document. For dated references, only the edition cited applies. For undated references, the latest edition of the referenced document (including any amendments or corrigenda) applies.

P1597.2 (Draft 2.0, 28 February 2007), Draft Recommended Practice for Validation of Computational Electromagnetics Computer Modeling and Simulation.

IEEE EMC Society's modeling web site

3. Acronyms

For the purposes of this standard, the following terms and definitions apply. Annex C, *The Authoritative Dictionary of IEEE Standards Terms, Seventh Edition* and ANSI C63.14-1998, should be referenced for terms not defined in this clause.

ADM	Amplitude Difference Measure (depending on context)
CEM	computational electromagnetics
EM	electromagnetic
EMC	electromagnetic compatibility
EMI	electromagnetic interference
EMI/C	electromagnetic interference/compatibility
FDM	Feature Difference Measure
FDTD	Finite-Difference Time-Domain
FEM	Finite Element Method
FSV	Feature Selective Validation
GDM	Global Difference Measure
Mil-Std	military standard
M&S	modeling and simulation
MoM	Method of Moments
OATS	open area test site
PCB	printed circuit board
PEC	perfectly electrically conducting
PEEC	Partial Element Equivalent Circuit
RAM	random access memory (depending on context)
RLC	resistance, inductance, and capacitance
RCS	radar cross section
TLM	Transmission Line Matrix
VRS	Validation Rating Scale

4. CEM modeling and simulation validation process

4.1 General

The validation process for M&S results is dependent on exactly what is being validated and what data is available to support the validation. The preferred process is to validate the unknown case against a known reference. To this end, standard problems and other means of obtaining a reference are described in this standard together with a procedure to compare the resulting data sets. Where no suitable reference case can be obtained, validation is possible by using self referencing schemes. While less encompassing, these schemes still permit a high level of confidence to be achieved. The flow chart shown in Figure 1 provides the route that is taken.

NOTE—The terms 'modeling' and 'simulation' can, in most places, be used interchangeably.

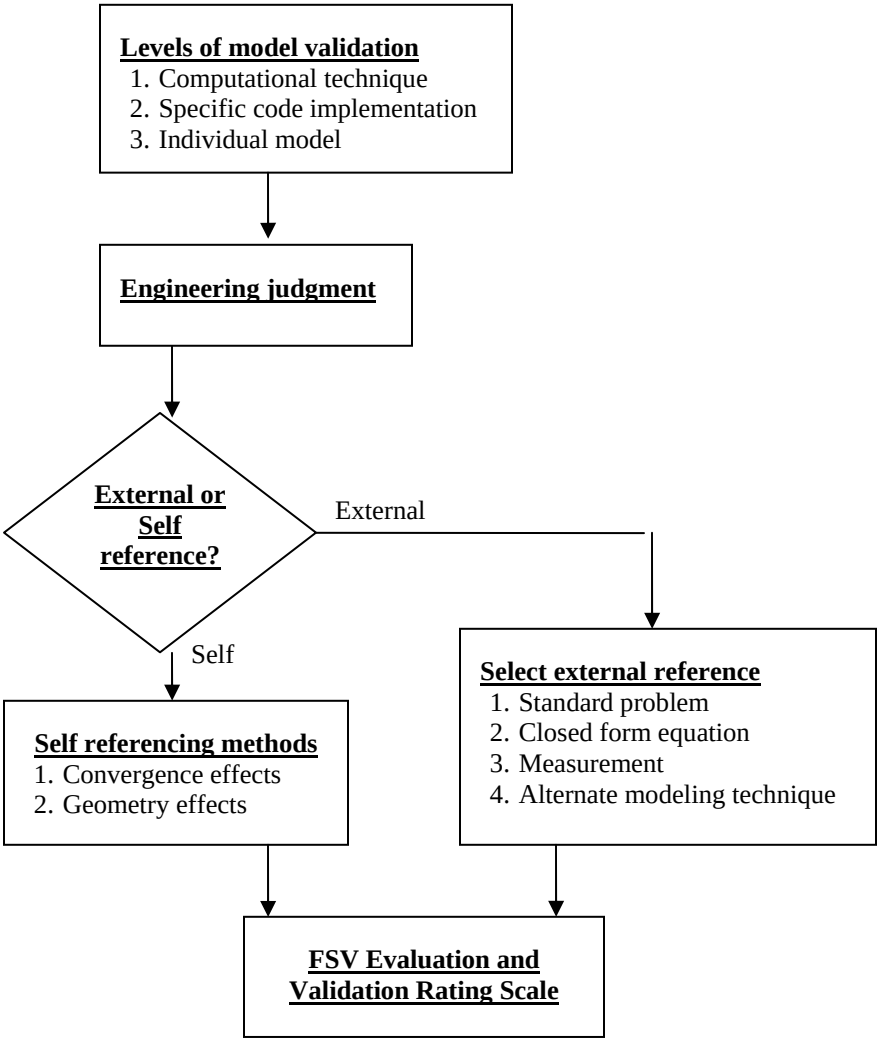

Figure 1 – Validation flow chart

4.2 Levels of model validation

There are three different levels to a complete model validation. When deciding how to validate a model, it is important to consider which levels of validation are needed. The levels are:

1. Mathematical level: Computational technique validation
2. Implementation level: Individual software code implementation validation
3. Model level: Specific model validation

The first level determines if the underlying computational technique is correct. The second expands this to include a particular code implementation, and the third includes the specific details of a model. For a complete model validation, all three of these levels are required to be correct.

NOTE—The term 'numerical' could be used equally in place of 'computational'. The authors of this document have chosen to use 'computational'.

4.2.1 Computational technique validation

The first level of model validation is the computational technique. This is usually unnecessary in most EM modeling problems since the computational technique will have been subject to extensive experimental validation. If a new technique is developed, it too must undergo extensive validation to determine its limitations, strengths, and accuracy. When the user is looking to validate a given code or a new computational technique then a wide range of models may be needed to fully bound the intended application.

4.2.2 Individual software code implementation validation

The second level of validation is to ensure the veracity of the software implementation of the computational technique, and ensure the creation of correct results for the defined model. It is prudent to test individual codes against the types of problems for which they will be used.

For example, a software vendor will have a number of different examples where their software code has been used, and where tests or calculations have shown good correlation with the modeled results. This helps the potential user to have confidence in that software code for those applications where there is good agreement. However, there may still be application related limitations in how the code is used and implemented. When a previous validation effort is to be extended to a new use, the types of problems that have been validated in the past must closely match the important features of the current model.

4.2.3 Specific model validation

The third level of validation is for a specific model, and is the most common concern for engineers. In nearly all cases, software modeling tools will provide a very accurate answer to the question that was asked. However, there is no guarantee that the correct 'question' was asked. That is, the user may have inadvertently specified a source or some other model element that does not represent the actual physical structure intended.

4.3 Preparation for validation using engineering judgment

Prior to performing any detailed validation analysis, the user shall apply engineering judgment on the solutions to ensure that the model being validated is basically sound. The following list is not comprehensive and other checks may be more appropriate for a given model; however, if any of the described cases apply, time may be saved if an error can be identified early.

1. Some of the different simulation techniques (such as MoM and PEEC) will calculate the currents over the entire structure. The radiated electric fields are determined from the currents. These

currents provide significant insight to the computational result's validity. Viewing the currents at specific frequencies, especially near resonance, permits the user to observe the standing wave patterns, behavior in the areas of discontinuities, and breaks in the metal surfaces. The currents should not vary rapidly in adjacent patches or segments, and should be near zero at the ends of wires or planes. If these requirements are not met, it indicates that there is a problem with the model.

2. When using time-domain simulation techniques (such as FDTD, PEEC, and TLM), the fields, currents, or voltages are found for all the cells within the computational domain for each time step. Typically, the final result desired is the field strength at a specific location or number of locations. However, viewing the fields, currents, or voltages as they propagate through the computational domain can provide significant insight to the computational result's validity.

3. While observing the fields within a volume-based, time-domain, simulation technique (such as FDTD and TLM), the user should observe the field animation to insure that the fields were not reflected from the computational boundary, they did propagate past all observation points, and all resonances were dampened to a sufficient point to indicate that the simulation is complete. Simply observing the final field results at some location is not a guarantee that all the above possible effects were properly simulated.

4. Time-domain simulation techniques that are current or voltage based (such as PEEC and TLM) are especially useful for circuit board models. The animation of the currents or voltages may be observed to ensure that the intentional signal propagates along the intended path, and that any resonances are dampened to a sufficient point to indicate that the simulation is complete.

5. Under some circumstances, it is possible to use known quantities to validate a model. For example, the radiation pattern of a half-wave dipole is a well known quantity and, if the model is similar to a half-wave dipole, a dipole pattern simulation will increase confidence in the simulation results from the primary model.

6. Often, simple sanity checks can be performed based on mathematical "boundary conditions". Examples include: checking the symmetry of the electromagnetic field solution in a symmetric problem set up, checking the consistency of the field solution with Maxwell's equations, and checking the consistency of the field solutions with derived equations (e.g. conservation of charge and energy).

4.4 External or self references

Once the simulation results to be validated are defined, the appropriate validation reference must be selected. Where possible, a complete solution will be compared to an external reference (see Section 5). Standard problems have been solved and are available to provide highly reliable external references. Where these are insufficient, a reference may be obtained from closed form solutions, measurements, or alternate modeling techniques.

When it is not possible to make a comparison, it is necessary to ensure that the model is accurate through other confidence building measures. These are self referenced procedures (see Section 6) and include convergence effects and problem geometries.

5. External references for model validation

5.1 General

To perform a validation with an external reference, it is necessary to select a suitable reference against which the computations can be compared. Suitable references may include:

1. Standard problems
2. Closed form equations
3. Measurements
4. Other modeling techniques

The selection of an appropriate reference is critical as it forms the basis for the validation effort. It is therefore essential that a reference of the highest possible accuracy is used.

5.2 Standard problem references

A number of standard validation problems have been proposed over the recent years [B6], [B7], [B8], [B9] to assist engineers who wish to evaluate the various vendor modeling tools against specific problems that are similar to the types of problems that they wish to simulate. A wide variety of problems have been addressed and the results are available on the IEEE EMC Society's modeling web site. Solutions exist for problems involving printed circuit boards, antennas, shielding, and benchmark problems. These are fully specified and can be used as the reference for model validation.

5.3 Closed form equation references

Selecting a representative case where the geometry is sufficiently simple to solve using closed form equations must be done with care. For example, an accurate simulation of the reflection from a perfectly conducting sphere can provide a good indication but no guarantee that simulations of reflections from a complex object, such as an airplane or tank, will produce a correct result. Also, that same set of equations provides little or no indication of how the code will predict printed circuit board emissions or shielding performance.

Closed form equations are generally specialized and may not represent the real-world problem of interest to the user. Therefore, closed form equation references must be selected with care to ensure they are appropriate to the situation.

5.4 Measurement references

When using measurements as the basis for validation, great care must be exercised to ensure that the measurement uncertainty is minimized. For example, antenna and RCS measurements may be made with high precision while those made for EMI/C purposes may have uncertainties exceeding 6 dB.

There are a number of issues that must be addressed to minimize measurement related validation errors.

1. The measurement environment geometry must be accurately represented in the model
2. Overall measurement accuracy must be understood
3. Loading effects of measurement equipment must be included

Care must be exercised in setting up the computational model, as is done for any precision field measurement. Cables and measurement facility effects must all be taken into full account in the model. Computer models can suffer from perfection, such as pure polarization, perfect alignment to the axes, and non invasive field probes, while real measurements must face an imperfect test environment. Failure to consider the real measurement configuration can lead to unnecessary errors. The goal is to ensure that the measured problem geometry and the modeled one are identical in every way.

The most common references for the validation of EMI/C models are actual measurements. This is largely due to the fact that EMI/C type problems do not lend themselves to closed-form calculations. The test environment (open area test site (OATS) vs. anechoic vs. semi-anechoic), antenna height, and antenna pattern will have a significant effect on the measurement that, if not included in the simulation, will cause

the results to differ. One of the properties of simulation is that a 'perfect' environment is easily created. While this allows the user to focus on the desired effects without consideration of the real world test environment, it is essential that the real environment be included in any model being validated against a measured reference.

Loading effects of the measurement system on the device under test must be included. For example, the input impedance of a spectrum analyzer or network analyzer must be included when used to measure values on a printed circuit board or the signal on a small probe is being modeled. While the 50 Ohm load of the analyzer does not necessarily represent the real-world environment that the PCB will be operating in, it becomes very important when a simulation is to be compared to a laboratory measurement. Antennas may need to be fully modeled rather than just including a field monitor point if there is tight coupling between the antenna and another part of the measured problem.

Measurement accuracy and repeatability must also be considered for a reference measurement. A detailed analysis of the measurement precision must be made; for antenna measurements this maybe less than a tenth of a dB, while for commercial EMI/C measurements it may exceed 6 dB. Mil-Std EMI/C laboratories are also considered to have a much higher measurement uncertainly when they use a plain, shielded room test environment.

The agreement between the modeled data and the measurement data can be no better than the test laboratory's uncertainty. If measurement data disagrees with modeled data, some consideration should be given to the possibility that the measurement was incorrect and the model data correct, or that the measured and modeled cases differed in some small but significant way. It is essential to avoid measurement bias and to understand both the measured and modeled cases equally.

5.5 Alternate modeling technique derived reference

Another popular approach to validating simulation results is to model the same problem using two different modeling techniques. If the physics of the problem are correctly modeled with both simulation techniques, then the results should agree. Achieving agreement from more than one simulation technique for the same problem can add confidence to the validity of the results.

There are a variety of full wave simulation techniques. Each has strengths, and each has weaknesses. Care must be taken to use the appropriate simulation techniques and to make sure they are different enough from one another to make the comparison valid. Comparing a differential equation based simulation technique (i.e. FDTD, FEM, and TLM) with an integral equation based technique (i.e. MoM and PEEC) is preferred because the very nature of the solution approaches are different. While this means that more than one modeling tool is required, with the added time and expense of developing expertise in using more than one tool, the value of having confidence in the simulation results is much higher than the cost of many vendor software tools.

By the very nature of full wave simulation tools, structure-based resonances often occur. These resonances are an important consideration to the validity of the simulation results. Most often, the simulations of real-world problems are subdivided into small portions due to memory and model complexity constraints. These small models will have resonant frequencies that are based on their arbitrary size, and have no real relationship to the actual full product. Results based on these resonances are often misleading, since the resonance is not due to the effect under study, but rather it is due to the size of the subdivided model. Care should be taken when evaluating a model's validity by multiple techniques to make sure that these resonances are not confusing the 'real' data.

6. Self references for model validation

6.1 General

When the model cannot be compared to an external or independent reference, a self referenced model validation shall be performed. This is a less reliable method as it is possible to carry over an error from the original, or primary model and so shall only be used when an external or independent reference is not possible.

Secondary, self reference models shall be created using the same approach as the primary model, and these secondary models will be used as the basis for validating the primary model. It is required that at least two variations of secondary models be created, although more are recommended to minimize the risk of missing a source of error.

There are two kinds of changes that can be made to the primary model parameters that will create valid secondary models; changes that should not substantially affect the result, and changes that will have a known effect on the solution. These are termed:

1. Convergence effects, which include:

    - problem grid, segment, or cell size
    - computational white space domain
    - absorbing boundary conditions

2. Geometry effects, which include:

    - size of apertures
    - number of apertures
    - component placement on PCBs

The results from the primary and secondary models shall be compared using the FSV procedures, as detailed in section 7.

6.2 Convergence based self referenced models

There are a number of model parameters that must be decided before the actual simulation can be performed. The size of the grid or cell is often set to a maximum of one tenth of a wavelength to satisfy the assumption that the currents or fields do not vary within each grid or cell. However, this size may not be small enough to correctly capture the currents or fields if their amplitude varies rapidly on the structure. Changing the size of the grid or cell is a good way to ensure that the proper size was used. If the grid or cell size is correct, the final results from the simulation will not change. However, if the results change when the grid or cell size is changed, then the correct size was not used.

With volume based techniques (such as FDTD, FEM, and TLM), the size of the computational domain can be varied. This will make sure there are no spurious responses or absorbing boundary mesh truncation effects that interact with the physical model. The final result should not be dependent on the size of the computational domain or the distance between the absorbing boundary mesh truncation and the physical model. Again, if the results are seen to change as these parameters are changed, the model was not correctly formed.

For cases where multiple absorbing boundary mesh techniques are available, it is appropriate to ensure that the correct one is chosen.

6.3 Geometry based self referenced models

This class of reference model uses changes in the original model to predict specific changes in the solution. Within a model, there are usually a number of parameters that are critical in obtaining the results. Size of apertures, number of apertures, and component placement on PCBs can vary the final result from the simulation. In many cases, the effect of changing a parameter can be predicted from experience, even though the actual amount of variation may not be known in advance.

For example, the size of the aperture can be increased, and the shielding effectiveness for the different aperture sizes examined for 'reasonableness'. Resonant frequencies of the aperture should be seen to vary as the size of the aperture varies, providing another opportunity to check the results from the simulation. An extension of this is to close all apertures and verify that there is no leakage from the model.

7. Numerical calculation of the validation rating using the Feature Selective Validation procedure

7.1 General

After selecting the validation reference, a method of evaluating that reference must be employed. The Feature Selective Validation (FSV) procedure shall be used to determine that agreement between the validation reference and the new simulation results is acceptable.

A number of different techniques have been used in the past to compare two sets of data. Simple subtraction of one data set from the second data set will show the differences, but is very limited as a true indication of the overall agreement between the two data sets if there is a slight offset between the data sets. Similarly, cross correlation has been used, but it is difficult to relate the results from cross correlation to what the human 'expert' would decide with a visual inspection of the data sets.

The FSV is a technique that combines an amplitude-based comparison with a feature-based comparison to give an overall better indication of the agreement between two sets of data. The FSV has been calibrated to match human 'expert' comparison for decisions that are somewhat subjective, but will attach labels describing the agreement such as 'excellent', 'very good', 'good', 'poor' etc.

The FSV technique shall be used to quantify the comparison of data sets for validation purposes. A minimum expectation is that the single value of the Global Difference Measure (GDM) along with the Grade and Spread of the GDM will be reported. Additional levels of detail, available from the FSV method, can be included as appropriate to assist in the comparison and in developing an understanding of any disparity between the data sets.

7.2 Feature Selective Validation (FSV)

Experienced professionals can look at the data in Figure 2 and decide that the two plots have 'good' agreement or, alternatively, decide that the comparison shows 'fair' agreement, depending upon their expectations and background. The FSV allows the comparison to be quantified objectively, removing the element of subjectivity from the decision making process.

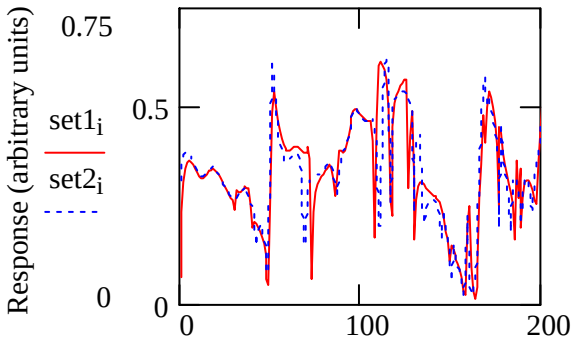

Figure 2 – Example of two data sets for comparison

The basis of the FSV technique is the decomposition of the results to be compared into two 'component' measures and then the recombination of the results to provide a global *goodness of fit* measure. The components used are the Amplitude Difference Measure (ADM) that compares the amplitudes and 'trends' of the two data sets, and the Feature Difference Measure (FDM) that compares the rapidly changing features (as a function of the independent variable). The ADM and FDM are then combined to form a Global Difference Measure (GDM). All of the ADM, FDM, and GDM are usable as point-by-point analysis tools or as a single, overall measurement.

7.3 Validation rating scale

The Validation Rating Scale (VRS) establishes a benchmarking technique, allowing 'visual' comparisons to be made between experimental and simulated data (or any two data sets). This technique provides the analyst with an overall quantitative measure of similarity. This section details the VRS proposed in COMPEL: International Journal for Computation and Mathematics in Electrical and Electronic Engineering [B21]. The purpose of the validation rating scale is twofold; first it allows individuals or groups of experts to visually assess a comparison using a common frame of reference. This is particularly useful to verify the agreement between FSV results and visual assessment. Secondly, it allows the translation of FSV output back to a visual basis.

Validation of electromagnetic modeling methods, particularly against experimental results, often involves structurally complex data sets. Quantification of these comparisons are usually made 'by eye', to determine how similar the two traces appear. This method of validation has obvious limitations. The subjectivity of this approach makes expressing the degree of similarity very difficult. Also, there is no absolute scale by which to make the comparisons. Hence rating the pairs of data depending on their similarity is a demanding task. The VRS provides the confidence that is lacking in the old visual approach.

The VRS is a 6-point scale requiring only a binary decision at each node, and is shown in Figure 3. The FSV technique removes the subjectivity when validating the data sets, by guiding the analyst through the comparison to be made. The VRS therefore solves the problem of the users having no common set of definitions with which to compare data sets. Qualitative, natural language descriptors have been allocated to these values. The descriptors used are Excellent, Very Good, Good, Fair, Poor, and Very Poor, reflecting a common comparative lexicon.

A group mean value can be obtained from a set of individual comparisons made using the VRS using Equation (5.1).

$$Mean\ Rating = \sum_{i=1}^{6} i.P_i \qquad (5.1)$$

where

i is the category (1 through 6)

P_i is the proportion of results in category i

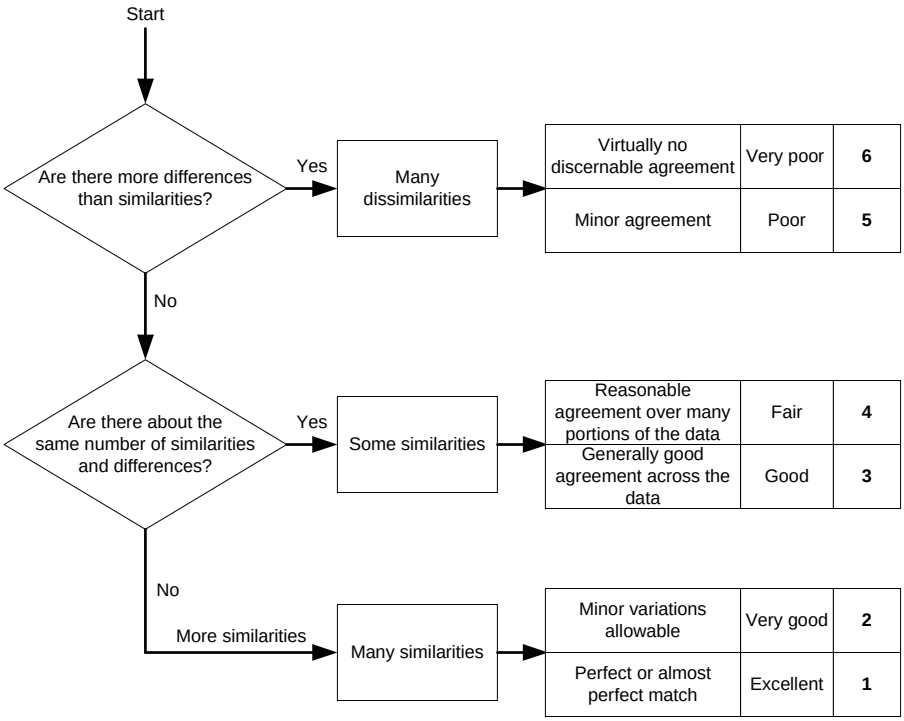

Figure 3 – Validation rating scale for visual benchmarking

7.4 Procedure

Calculation of the ADM and FDM requires access to a Fourier Transform capability. The procedure is presented hereafter, and is as found in *Feature Selective Validation (FSV) for validation of computational electromagnetics (CEM). Part I – The FSV method* [B24].

The procedure assumes that two separate data sets are available for comparison. Both data sets share a common independent variable (plotted as an x-axis) and a common dependent variable (plotted as a y-axis). The two data sets may:

- contain different numbers of data points;
- exist over different ranges of the common independent variable;
- have either coincident or non-coincident values of the common independent variable.

The procedure refers to these data sets as the input data sets. These are manipulated during the procedure to generate the working data sets, i.e. the data sets used in the calculation of the ADM and FDM.

1. Generation of the Working Data Sets
   This step sees the input data sets manipulated to create the working data sets. The working data sets contain the same number of data points and have coincident values of the common independent variable.

   a. Compare the input data sets to establish the region of overlap within the common independent variable;

   b. Identify those data points within each data set that exist within the region of overlap;

   c. Identify the number of data points within each data set that exist within the region of overlap;

   d. For the input data set with the smallest number of data points within the region of overlap, extract the data points within the region of overlap to form the first working data set;

   e. Copy the values of the independent variable from the first working data set to the second working data set;

   f. Use the remaining input data set (that with the highest number of data points within the region of overlap), apply linear interpolation within the region of overlap to generate the value of the independent variable at each coincident value of the second working data set;

   Note that when both input data sets have the same number of data points within the region of overlap, either data set may be used in step d, above.

2. Fourier Transform both working data sets. This creates the transformed working data sets.

3. The following procedure is to be performed for both transformed working data sets:

   a. Perform an Inverse Fourier Transform on the content of the first four data points within the transformed data set. The vectors returned shall be labeled as $DC_s(n)$, where the subscript, s, is either 1 or 2 and denotes the transformed working data set number.

   b. Calculate the sum of the values, S, of the independent variable, starting at the fifth data point and continuing to the end of the data set. Thus:

$$S = \sum_{i=5}^{N} TWDS(i)$$

   where

S	is the sum of the values of the independent variable;
i	is the data set element;
N	is the total number of elements within the data set;
$TWDS(i)$	is the ith independent variable within the transformed data set.

   c. Determine the ‘40%’ location. This is the *i*th data point, at which the sum of the values of the independent variable, starting at the fifth data point and continuing to the *i*th data point, is greater than or equal to 40% of the value calculated during step (b). The value of i is systematically increased from the fifth data point until this condition is reached. Thus:

$$\sum_{i=5}^{i_{40\%}} TWDS(i) \geq 0.4 \cdot S$$

   where

S	is the sum of the values of the independent variable;
i	is the data set element;
$TWDS(i)$	is the value of the ith independent variable within the transformed data set;
$i_{40\%}$	is the element containing the '40%' location.

d. Determine the 'break-point' location. This is five data points higher than the '40% location' determined in step (c), above. Thus:

$$i_{40\%} = i_{40\%} + 5$$

where

$i_{40\%}$	is the element number of the 40% location;
i_{bp}	is the element number of the 'break point'.

e. Apply the filter displayed as Table 1 to the independent variable values contained within the indicated elements of the transformed working data set to generate the filtered transformed working data set.

Table 1 – Filter Definition – a

Element Number	Filter Value
I_{bp}-3	*1.000*
I_{bp}-2	*0.834*
I_{bp}-1	*0.667*
I_{bp}	*0.500*
I_{bp}+1	*0.334*
I_{bp}+2	*0.167*
I_{bp}+3	*0.000*

f. Perform an Inverse Fourier Transform on the filtered transformed working data set. The vectors returned shall be labeled as $Lo_s(n)$, where s is either 1 or 2 and denotes the data set number.

4. The following procedure is to be performed for both transformed working data sets:

a. Apply the filter displayed as Table 2 to the independent variable values contained within the indicated elements of the transformed working data set to generate the filtered transformed working data set.

Table 2 – Filter Definition – b

Element Number	Filter Value

I_{bp}-3	0.000
I_{bp}-2	0.167
I_{bp}-1	0.334
I_{bp}	0.500
I_{bp}+1	0.667
I_{bp}+2	0.834
I_{bp}+3	1.000

b. Perform an Inverse Fourier Transform on the filtered transformed working data set. The vectors returned shall be labeled as $Hi_s(n)$, where s is either 1 or 2 and denotes the data set number.

5. For each set of data points within the Working Data Sets, the ADM is to be calculated using Equation (1).

$$ADM(n) = \left|\frac{\alpha}{\beta}\right| + \left|\frac{\chi}{\delta}\right| \exp\left\{\left|\frac{\chi}{\delta}\right|\right\} \quad (1)$$

where

$$\alpha = \left(\left|Lo_1(n)\right| - \left|Lo_2(n)\right|\right)$$

$$\beta = \frac{1}{N}\sum_{i=1}^{N}\left(\left(\left|Lo_1(i)\right| + \left|Lo_2(i)\right|\right)\right)$$

$$\chi = \left(\left|DC_1(n)\right| - \left|DC_2(n)\right|\right)$$

$$\delta = \frac{1}{N}\sum_{i=1}^{N}\left(\left(\left|DC_1(i)\right| + \left|DC_2(i)\right|\right)\right)$$

n is the nth data point

6. Calculate the mean value of ADM from Equation (2).

$$ADM = \frac{\sum_{n=1}^{N} ADM(n)}{N} \quad (2)$$

where
The mean value of the ADM(*n*) gives a single-figure measure of 'goodness-of-fit'

7. Calculate the ADM confidence histogram. The range of values for the ADM and, in fact, the FDM and GDM can be divided into six categories, each with a natural language descriptor: Excellent, Very Good, Good, Fair, Poor, and Very Poor. These are the terms that are most often used in descriptions of the quality of comparisons. The confidence histogram, like a probability density function, provides some intelligence as to how much emphasis can be placed on the single figure of merit. There is some evidence to show that this mirrors the overall group assessment of

any data pair by a number of engineers. The determination of the histogram is simply a case of counting the proportion of points that fall into one of the categories, according to the rule base in Table 3.

Table 3 – FSV interpretation scale

FSV value (quantitative)	FSV interpretation (qualitative)
Less than 0.1	Excellent
Between 0.1 and 0.2	Very good
Between 0.2 and 0.4	Good
Between 0.4 and 0.8	Fair
Between 0.8 and 1.6	Poor
Greater than 1.6	Very poor

8. Calculate derivatives in preparation for the FDM calculation. The following components need to be calculated:

   a. The first derivatives of the Lo(*f*) and Hi(*f*) data sets

   b. The second derivatives of the Hi(*f*) data sets.

   The derivatives accentuate the high rate-of-change features in the original data, and differences based on the derivatives are combined in the determination of the FDM. The first derivatives are obtained using a central difference scheme as in Equation (3).

$$\text{Lo}'(f) = \text{Lo}(f + N_d) - \text{Lo}(f - N_d) \tag{3}$$

   where
   $N_d = 2$ for the first derivative

   The second derivatives of the Hi data sets are obtained from the first derivatives using a similar approach again, found in Equation (4)

$$\text{Hi}''(f) = \text{Hi}'(f + N_d) - \text{Hi}'(f - N_d) \tag{4}$$

   where
   $N_d = 3$

   Equation (5) shows the derivative operators (from Equation (3) and Equation (4)) in terms of templates that 'slide across' the original data to generate the first and second derivatives.

$$\begin{aligned} Hi' &= \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \end{bmatrix}^T \\ Hi'' &= \begin{bmatrix} 1 & 0 & 0 & 0 & -1 & 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}^T \end{aligned} \tag{5}$$

   where
   superscript *T* stands for transpose

9. Calculate the point-by-point FDM. The FDM is formed from three parts based on the derivatives calculated in step 8. The numerical values in the equations are part of the heuristic and have been determined empirically. Note that because the following equations describe point-by-point values in the numerator, the denominator is based on a mean-value approach, similar to the determination of the ADM. It should be noted that the combination of Equations (6), Equation (7), and Equation (8) results in Equation (9) resolving to the difference between a function of Dataset 1 and a function of Dataset 2.

$$FDM_1(f) = \frac{\left|Lo_1'(f)\right| - \left|Lo_2'(f)\right|}{\frac{2}{N}\sum_{i=1}^{N}\left(\left(\left|Lo_1'(i)\right| + \left|Lo_2'(i)\right|\right)\right)} \tag{6}$$

$$FDM_2(f) = \frac{\left|Hi_1'(f)\right| - \left|Hi_2'(f)\right|}{\frac{6}{N}\sum_{i=1}^{N}\left(\left(\left|Hi_1'(i)\right| + \left|Hi_2'(i)\right|\right)\right)} \tag{7}$$

$$FDM_3(f) = \frac{\left|Hi_1''(f)\right| - \left|Hi_2''(f)\right|}{\frac{7.2}{N}\sum_{i=1}^{N}\left(\left(\left|Hi_1''(i)\right| + \left|Hi_2''(i)\right|\right)\right)} \tag{8}$$

$$FDM(f) = 2\left(\left|FDM_1(f) + FDM_2(f) + FDM_3(f)\right|\right) \tag{9}$$

10. Calculate the single value of FDM. This is done in exactly the same way as for the ADM.
11. Calculate the FDM confidence histogram. This is done in exactly the same way as was done for the ADM.
12. Obtain the point-by-point GDM value. The GDM is premised on the ADM and FDM being largely independent, which means that:

$$GDM(f) = \sqrt{ADM(f)^2 + FDM(f)^2} \tag{10}$$

13. Calculate the overall GDM value and the GDM confidence histogram. This follows the same procedure as the ADM and FDM.
14. Determine the equivalent visual scale values for ADM, FDM, and GDM. As discussed previously, the FSV values can be categorized into a six point, natural language scale. The VRS is presented in section 7.3. This is done using a piecewise conversion. The piecewise approach for this is given in Table 4, where X is the ADM, FDM, or GDM (total or point-by-point), and V is the transformed (visual) scale equivalent.

Table 4 – Piecewise visual conversion

If $X \leq 0.1$, then $V = 1 + 10X$
If $X > 0.1$ and $X \leq 0.2$, then $V = 2 + 10 (X – 0.099)$
If $X > 0.2$ and $X \leq 0.4$, then $V = 3 + 5 (X – 0.199)$
If $X > 0.4$ and $X \leq 0.8$, then $V = 4 + 2.5 (X – 0.399)$
If $X > 0.8$ and $X \leq 1.6$, then $V = 5 + 1.25 (X – 0.799)$
If $X > 1.6$, then $V = 6$

7.5 Grade and Spread

The GDM previously described was obtained by treating the ADM and FDM as equally important. The representative nature of the FSV will be improved by weighting the ADM and FDM in the GDM calculation based on:

- How concentrated the histogram bars are at one extreme or the other (Grade).
- How dispersed the histogram bars are around the mean value (Spread).

This information will be obtained by determining the ‘Grade’ and the ‘Spread’ of the ADM and FDM, that will be used to weight the components and can be displayed graphically. The Grade gives a numerical indication of the quality of the comparison and the Spread gives a numerical indication of the level of confidence that can be placed on this assessment. The Grade is given a numerical value by counting how many categories are required (starting from Excellent) for the cumulative total of the histogram to exceed a given value. A numerical value is given to the Spread by counting how many adjacent categories (starting from the largest) are required to cumulatively exceed a given value. It emphasizes that a wide Spread gives a poorer overall quality of result than a wider Grade. A default value for both the Grade and Spread is suggested to be 85 %, but this may be varied according to requirements.

Annex A

(informative)

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Annex B

(informative)

Guidelines on the selection of CEM techniques and codes

B.1 Ensemble of problem drivers

Applying the right techniques to the CEM M&S task involves a number of important decisions. Without some knowledge and experience of the CEM discipline, it is not intuitively obvious which techniques or codes are best suited to solving a particular problem. The choice depends on several factors. These include the appropriate physics to be applied, the category of problem to be solved, desired accuracy, and the "observables" to be computed. Observables here refer to surface currents, scattered fields, or other figures of merit that are used to "measure" the electromagnetic phenomenon or effect. Although high accuracy is nearly always desirable, it may be computationally expensive due to the potentially significant computer resources that may be required to solve the problem. This, in turn, influences not only how the physical problem of interest is modeled to begin with, but also what solver will be used to compute the observable(s) of concern. Clearly, the types of physics and solution methods applied to a given problem, as well as the desired observables and accuracy requirements are all inter-related and affect the downstream M&S process. In fact, there are other factors that can significantly contribute to the success or failure of the M&S approach, such as the set of modeling primitives (i.e., building blocks) that are used to represent the physical problem of interest. A representation of the concept and decision scheme involved in selecting the appropriate solution method for validity and consistency is illustrated in Figure B.1.

Various types of computer software codes are available for modeling, simulating, and analyzing complex electromagnetics problems. Regardless of the code or computational method used, the applied engineering theories and physics formalisms are based fundamentally on the integral or differential forms of Maxwell's equations. As described above, the selection of the appropriate CEM modeling technique or code for a given problem is driven by several criteria. These are:

(a) the type and complexity of the problem to be solved (i.e., computing gross, system-wide, coupling interactions for large complex structures versus analyzing "local" interference associated with small circuits or device ports);

(b) the desired modeling accuracy or analysis fidelity versus inherent limitations of the physics, modeling technique, or software code; and

(c) whether the problem to be solved contains open or closed geometries and boundaries, or other modeling nuances which may either call for or eliminate certain techniques.

These criteria are not all inclusive. Indeed, there are additional factors that need to be considered, on a case-by-case basis, in selecting one type of computational technique over another. We will not focus here on all of the various factors and considerations that may arise in deciding which computational method to

apply as oftentimes the final decision is system, problem, or applications specific, and may be influenced by the analyst's preferred technique(s) or code(s) of choice.

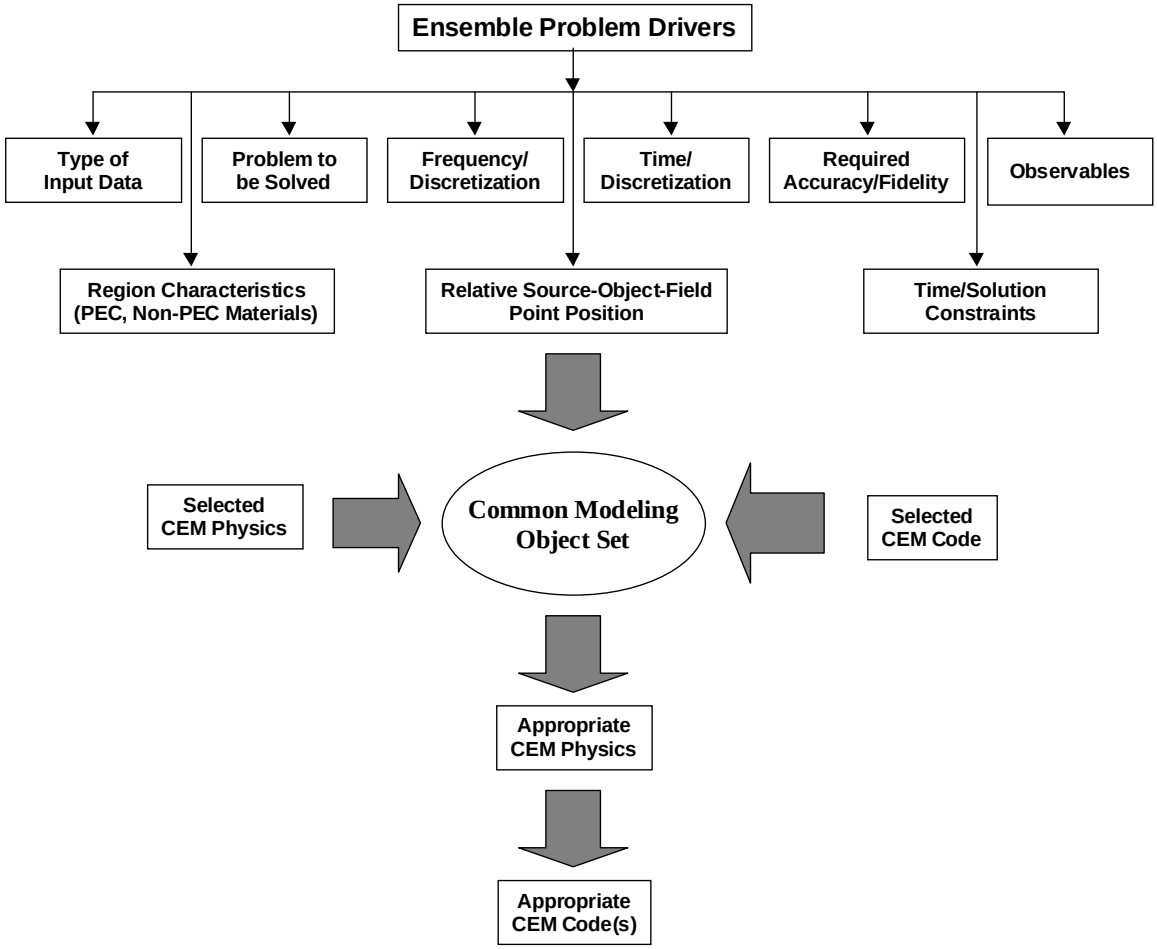

Figure B.1--Problem "drivers" influencing the CEM modeling, simulation, and validation approach

B.2 Categories of CEM techniques

This section provides an overview of the different categories of CEM techniques. Several of the more commonly used techniques are briefly discussed for background purposes. However, this annex does not attempt to present a detailed treatise on the subject.

NOTE—This document intentionally adopts a *code agnostic* position with regard to specific CEM codes i.e., the discussions do not cite, endorse, or recommend any specific CEM codes available from Government, academic, or commercial industry sources for neutrality and objectivity purposes.

Computer methods for analyzing problems in EM generally fall into one of three categories: analytical techniques, expert systems, and computational techniques.

Analytical techniques make simplifying assumptions about the geometry of a problem in order to apply a closed-form (or table look-up) solution. A number of computer programs based on analytical techniques are available to the EM engineer. Some are very simple and run on personal computers. Others are very elaborate and large scale. Analytical techniques can be a useful tool when the important EM interactions of the configuration can be anticipated. However, most EMC problems of interest are simply too unpredictable to be modeled using this approach.

Expert systems do not actually calculate the field directly, but instead estimate values for the parameters of interest based on a rules database. Expert systems approach a problem in much the same way as a quick-thinking, experienced, EM engineer with a calculator would approach it. As system design and board layout procedures become more automated, expert system EM software will certainly play an important role. Nevertheless, expert systems are no better than their rules database and it is unlikely that they will ever be used to model or understand the complex EM interactions that cause electromagnetic interference (EMI) sources to radiate.

Computational techniques attempt to solve fundamental field equations directly, subject to the boundary constraints posed by the geometry. Computational techniques generally require more computation than analytical techniques or expert systems, but they are very powerful EM analysis tools. Without making *a priori* assumptions about which field interactions are most significant, computational techniques analyze the entire geometry provided as input. They calculate the solution to a problem based on a full-wave analysis.

A number of different computational techniques for solving EM problems are available. Each computational technique is well-suited for the analysis of a particular type of problem. The computational technique used by a particular EM analysis program plays a significant role in determining what kinds of problems the program will be able to analyze.

The following sections review the strengths and limitations of different computational techniques for analyzing electromagnetic configurations. A particular emphasis is placed on how these techniques could be applied to the analysis of EMI sources.

B.3 Categories of computational techniques

Broadly speaking, CEM techniques can be subdivided into two basic categories: frequency-domain techniques and time-domain techniques. These can be further expressed as either integral or discrete formulations of Maxwell's equations. Essentially, solutions to these equations involve a series of partial differential equations that are subject to boundary constraints, except for some variations that are particular to the physics of a given problem. For instance, a CEM technique can be used to solve the Laplace equation that describes the potential distribution of a closed boundary. Also, a CEM technique can be applied to solving the Helmholtz wave equation, which arises in many electromagnetic radiation problems in open space. The point here is that there are different techniques and formulations for different problem solving applications. When and which technique(s) to apply and how accurate the solutions will be are the central questions.

More specifically, these categories can be further subdivided into methods that deal with system topologies represented by Perfectly Electrically Conducting (PEC) smooth surfaces, or meshes consisting of elemental thin wires, small n-sided polygonal patches or facets. One can extend and subdivide these methods even further by taking into account certain additional modeling constraints, boundaries, and topology variances that involve bounded or unbounded surfaces and the use of dissimilar materials. These additional constraints include:

- electrical size;
- specification of loading or non-PEC dispersive layers and other dielectric material properties (homogeneity, isotropy);

- conductor shapes and number of material layers;
- closed or open geometries including waveguide-like structures;
- existence of static or quasi-static field sources;
- steady state versus transient excitations and responses; and
- other topological, material, and electrical factors.

While each of these methods presents certain advantages and in some cases possible disadvantages, depending on the above considerations, these can be applied judiciously in an individual manner or in combination in order to study different aspects of a given electromagnetic problem.

So there exists a fairly broad range of diverse, but complementary computational methods available to the EMC analyst. No single modeling technique will be the most efficient and accurate for every possible model needed. Many commercial packages specialize in only one technique, a side effect that obligates the user to force every problem into a particular solution technique. The EMI/C engineer has as wide variety of problems to solve, requiring an equally wide set of tools. The “right tool for the right job” approach applies to EMI/C engineering as much as it does to building or designing any complex system.

Different modeling techniques are suited to different problems.

CEM techniques based on integral equations are advantageous in systems where electromagnetic waves are radiated in open regions. The equations are usually discretized using the Method of Moments (MoM) in which an unknown physical quantity is expanded in terms of a set of known expansion functions. This leads to a linear system of equations with N unknowns, where N is the number of expansion functions. The computational requirements are then at least proportional to N^2 in terms of both computer time and memory. Since N may be very large, these requirements are beyond the capabilities of today's computers (e.g., 76 GByte RAM if N=100,000). There are two possible ways to overcome this bottleneck: reduce the computational requirements to something lower than N^2, or simply reduce N (without sacrificing the accuracy).

There being a variety of electromagnetic modeling techniques, choosing the “best” technique is cause for a significant amount of debate, and often becomes a matter of past experience and personal choice or is driven by what tools are immediately available within the “toolbox”. Many of the techniques are specialized for certain configurations, and require cumbersome tailoring when used for each problem. Some techniques are not particularly generic, and require in-depth knowledge of electromagnetics and the computational modeling technique. Still others are useful only for far-field problems, such as determining a radar cross section of a piece of military equipment.

Annex C

(informative)

Definitions of CEM techniques

C.1 Adaptive Integral Method (AIM) - The AIM technique assists in making the iterative solution process more efficient and to speed up the matrix-vector multiplication.

C.2 Analytical Closed-Form Techniques - This method includes the use of non-numeric and quasi-discrete formulations that provide useful approximations and conservative or bounded analysis results. Most computational methods based on this approach are characterized in the frequency-domain. Many formulations in this category compute results based on real-valued (magnitude) quantities as opposed to computing complex magnitude and phase.

C.3 Bi-Conjugate Gradient Method with Fast Fourier Transform (BCG-FFT) - BCG-FFT techniques are useful in applications such as scattering and RCS, transient electromagnetic problems, the inverse problem, frequency selective surfaces, and optimum array processing. It considers Floquet's theory and the treatment of periodic conducting patches located in free space. Variations of this technique employ signal processing algorithms to arrive at solutions. BCG-FFT techniques are particularly useful in solving complex matrix equations generated by FEM/MoM methods.

C.4 Boundary Element Method (BEM) - BEM (like the FEM method) originated in the field of structural mechanics. It is a weighted residual technique that solves partial differential equations (PDEs) using mesh elements. It is essentially a MoM technique whose expansion and weighting functions are defined only on a boundary surface i.e., only the boundary of the domain of interest requires discretization. If the domain is either the interior or exterior to a volume, then only the surface is divided into mesh elements. The computational advantages of the BEM over other methods can be considerable. It is particularly useful for low frequency problems. The approach uses the simplest elements, is relatively easy to apply, and is versatile and efficient. BEM is mostly applicable to DC (electrostatics) and steady-state AC (electromagnetics) problems. Most general-purpose MoM modeling codes employ a boundary element method. Electrical engineers tend to use the more general term moment method to describe an implementation of this technique. Outside of electrical engineering however, the terms boundary element method or boundary integral element method are commonly used.

C.5 Conjugate Gradient Method (CGM) - The CGM technique is also based on the method of weighted residuals. It is very similar conceptually to conventional MoM techniques. Two features generally distinguish this technique from other moment methods. The first deals with the way in which the weighting functions are utilized. The second involves the method of solving the system of linear equations. Conventional moment methods define the inner product of a weighting function with another function *(*referred to as the *symmetric product*). The CGM technique uses a different form of the inner product called the *Hilbert inner product* that involves complex conjugation of real and complex weighting functions. When complex weighting functions are utilized, the symmetric product is a complex quantity and therefore not a valid *norm*. In this case, the Hilbert inner product is preferred. The other major difference between conventional moment methods and the CGM involves the technique used to solve the large system of equations these methods generate. Conventional moment method techniques generally employ a Gauss-Jordan method or another direct solution procedure. Direct solution techniques solve the

system of equations with a given number of calculations (N terms or unknowns). CGM utilizes an iterative solution procedure. This procedure, called the *method of conjugate gradients*, in conjunction with the *method of steepest decent*, can be applied to the system of equations or it can be applied directly to an operator equation. Iterative solution procedures such as the method of conjugate gradients are most advantageous when applied to large, sparse matrices.

C.6 Fast Multi-Pole Method (FMM) - FMM is a tree code-based method that uses two representations of the potential field: far field (multipole) and local expansions. The two representations are referred to as the "duality principle." This method uses a very fast calculation of the scalar potential field, which is computationally easier than dealing with the force vector (i.e., the negative of the gradient of the potential). The strategy of the FMM is to compute a compact expression for the potential phi (x, y, z), which can be easily evaluated along with its derivative, at any point. It achieves this by evaluating the potential as a "multipole expansion," a kind of Taylor expansion, which is accurate when $x^2+y^2+z^2$ is large.

C.7 Finite-Difference Frequency-Domain (FDFD) - Although conceptually the FDFD method is similar to the FDTD method, from a practical standpoint it is more closely related to the FEM method. Like FDTD, this technique results from a finite difference approximation of Maxwell's curl equations. However, in this case the time-harmonic versions of these equations are employed. Since there is no time stepping, it is not necessary to keep the mesh spacing uniform. Therefore, optimal FDFD meshes generally resemble optimal finite element meshes. Like the MoM and FEM techniques, the FDFD technique generates a system of linear equations. The corresponding matrix is sparse like that of the FEM method. Although it is conceptually much simpler than the FEM method, very little attention has been devoted to FDFD and very few available codes utilize this technique.

C.8 Finite-Difference Time-Domain (FDTD) - This method solves PDEs using a gridding technique with respect to a given boundary condition. In time-dependent PDEs, the FD method may be used in both space and time (i.e., FDTD) or it may be used for the spatial displacement component only for a given frequency (FDFD). FDTD techniques also require the entire volume to be meshed. Normally, this mesh must be uniform, so that the mesh density is determined by the smallest detail of the configuration. Unlike most FEM and MoM techniques, FDTD techniques are very well suited to transient analysis problems. Time stepping is continued until a steady state solution or the desired response is obtained. At each time step, the equations used to update the field components are fully explicit. No system of linear equations must be solved. The required computer storage and running time are proportional to the electrical size of the volume being modeled and the grid resolution. Because the basic elements are cubes, curved surfaces on a scatterer must be *staircased*. For many configurations this does not present a problem. However for configurations with sharp, acute edges, an adequately staircased approximation may require a very small grid size (0.1-0.25λ edge dimensions). This can significantly increase the computational size of the problem. Surface conforming FDTD techniques with non-rectangular elements have been introduced to alleviate this problem. Like FEM, the FDTD methods are very good at modeling complex inhomogeneous configurations. Also, many FDTD implementations do a better job of modeling unbounded problems than FEM codes. As a result, FDTD techniques are often the method of choice for modeling unbounded complex inhomogeneous geometries.

C.9 Finite Element Method (FEM) - This method, originating in the structural mechanics engineering discipline, solves PDEs for complex, nonlinear problems in magnetics and electrostatics using mesh elements. FEM techniques require the entire volume of the configuration to be meshed as opposed to surface integral techniques that only require the surfaces to be meshed. However, each mesh element may have completely different material properties from those of neighboring elements. In general, FEM techniques excel at modeling complex inhomogeneous configurations. However, they do not model unbounded radiation problems as effectively as MoM techniques. The method requires the discretization of the domain into a number of small homogeneous sub-regions or mesh cells and applying the given boundary condition resulting in field solutions using a linear system of equations. The model contains information about the device geometry, material constants, excitations and boundary constraints. The elements can be small where geometric details exist and much larger elsewhere. In each finite element, a simple (often linear) variation of the field quantity is assumed. The corners of the elements are called nodes. The goal of the FEM is to determine the field quantities at the nodes. Most FEM methods are

variational techniques that minimize or maximize an expression that is known to be stationary about the true solution. Generally, FEM techniques solve for the unknown field quantities by minimizing an energy quantity. It is applicable to a wide range of physical/engineering problems and frequencies, provided it can be expressed as a PDE.

C.10 Finite Integration Technique (FIT) - Unlike the FDTD method, which uses the differential form of Maxwell's equations, the FIT discretizes Maxwell's equations written in their original (integral) form, on a 3D domain. The unknowns are thus electric voltages and magnetic fluxes, rather than field components along the three space directions. Like all full 3D methods (FEM, FDTD, TLM, etc.), the entire 3D domain needs to be meshed. For Cartesian grids however, a special technique called Perfect Boundary Approximation (PBA) eliminates the staircase approximation of curved boundaries, for both PEC/dielectric and dielectric/dielectric interfaces. It allows even strongly non-uniform meshes, thus maintaining a manageable computational size. The FIT can be applied in both time domain (as the FDTD), and frequency domain (like FEM), on Cartesian, non-orthogonal-hexahedral, or tetrahedral grids. In the time domain, the explicit formulation leads to small memory requirements, and allows solving very large problems. From the time domain results, broad-band, high-resolution frequency-domain quantities are obtained by DFT, virtually at no extra cost. If the FIT is used directly in the frequency domain, the resulting matrices are sparse. The FIT is applicable to a variety of electromagnetic problems: in bounded or unbounded domains, for electrically small or very large structures, in inhomogeneous, lossy, dispersive, or anisotropic materials. It performs well from DC up to the THz region.

C.11 Finite-Volume Time-Domain (FVTD) - This technique, an extension of the FDTD approach, permits each element in the grid to have an arbitrary shape. Frequency domain results are obtained by applying a discrete Fourier transform to the time domain results. This requires additional computation, but a wide-band frequency-domain analysis can be obtained by transforming the system's impulse response. The FVTD (and FDTD) methods are widely used for RCS analysis although they have been applied to a wide range of EM modeling problems. Flexibility is their primary advantage. Arbitrary signal waveforms can be modeled as they propagate through complex configurations of conductors, dielectrics, and lossy non-linear, non-isotropic materials. Another advantage is that they are readily implemented on massively parallel computers, particularly vector processors and single-instruction-multiple-data machines. The only significant disadvantage is that the problem size can easily become unwieldy for some configurations. Grid resolution is generally determined by the dimensions of the smallest features to be modeled. The volume of the grid must be large enough to encompass the entire object and most of the *near field*. Large objects with regions containing small, complex geometries may require large, dense grids. When this is the case, other numerical techniques may be much more efficient than the FVTD (or FDTD) methods.

C.12 Generalized Multi-pole Technique (GMT - Moment Method) - GMT is a relatively new method for analyzing EM problems. It is a frequency-domain technique that (like MoM) is based on the method of weighted residuals. This method is unique in that the *expansion* functions are analytic solutions of the fields generated by sources located some distance away from the surface where the boundary condition is being enforced. MoM generally employs expansion functions representing quantities such as charge or current on a boundary surface. GMT expansion functions are spherical wave field solutions corresponding to *multi-pole* sources. By locating these sources away from the boundary, the field solutions form a smooth set of expansion functions on the boundary and singularities on the boundary are avoided. Like MoM, a system of linear equations is developed and solved to determine the coefficients of the expansion functions yielding the best solution. Since the expansion functions are already field solutions, it is not necessary to do any further computation to determine the fields. Conventional MoM methods determine the currents and/or charges on the surface first and then integrate these quantities over the entire surface to determine the fields. This integration is not necessary at any stage of the GMT solution. There is little difference in the way dielectric and conducting boundaries are treated by the GMT. The same multi-pole expansion functions are used. For this reason, a general purpose implementation of the GMT models configurations with multiple dielectrics and conductors much more readily than a general purpose MoM technique. On the other hand, MoM techniques, which employ expansion functions that are optimized for a particular type of configuration (e.g., thin wires), are generally much more efficient at modeling that specific type of problem. Over the last ten years, the GMT has been applied to a variety of EM configurations including dielectric bodies, obstacles in waveguides, and scattering from perfect conductors. Work and new

developments in this new method are continuing. Recent significant developments include the addition of a thin-wire modeling capability and a “ringpole” expansion function for modeling symmetric structures.

C.13 Geometrical Optics (GO) - The GO method applies exact ray tracing methods for light wave propagation through optical media. The method takes into account refraction, reflection and edge aberration phenomena and effects.

C.14 Geometrical/Uniform Theory of Diffraction (GTD/UTD) - UTD is an extension of the GTD method. These are high-frequency ray tracing techniques. They are only accurate when the dimensions of objects being analyzed are electrically large i.e., relative to the wavelength of the field. In general, as the wavelengths of an electromagnetic excitation approach zero, the fields can be determined using geometric optics (GO). UTD and GTD are extensions of the GO method combining the effects of direct rays, reflection, diffraction and multi-path propagation around an electromagnetic structure comprised of canonical elements. Diffraction is a local phenomenon at high frequencies. Therefore, the behavior of the diffracted wave at edges, corners, and surfaces can be determined from an asymptotic form of the exact solution for simpler canonical problems. For example, the diffraction around a sharp edge is found by considering the asymptotic form of the solution for an infinite wedge. GTD and UTD methods add diffracted rays to GO rays to obtain an improved estimate of the exact field solution. Canonical problems include those that are comprised of simple objects such as right circular or elliptical cylinders with end caps, ellipsoids, N-sided plates, cones, frusta, and spheres. Certain formulations allow canonical geometry modeling elements to be of the non-PEC type.

C.15 Hybrid Lumped Circuit and Quasi-Transmission Line Method - This is a hybridization of lumped circuit and TLM methods. The TLM formulation is appropriately modified to account for lumped circuit characterizations in 3-D microwave applications.

C.16 Hybrid Techniques - No one technique is well suited to all (or even most) electromagnetic modeling problems. Most MoM codes will not model inhomogeneous, nonlinear dielectrics. Finite element codes cannot efficiently model large radiation problems. GMT and UTD codes are not appropriate for small, complex geometries or problems that require accurate determination of the surface and wire currents. Unfortunately, most practical printed circuit card radiation models have all of these features and therefore, cannot be analyzed by any of these techniques. One solution, which has been employed by a number of researchers, is to combine two or more techniques into a single code. Each technique is applied to the region of the problem for which it is best suited. The appropriate boundary conditions are enforced at the interfaces between these regions. Normally a surface integral technique such as the boundary element method will be combined with a finite method such as the FEM, FDTD, or TLM method. Several successful implementations of hybrid techniques are described in the literature. So far, none of the available hybrid techniques model the radiation from printed circuit cards very well. This is due to the fact that most of these methods were developed to predict radar cross section (RCS) values or for other scattering problems where the source is remote from the configuration being modeled. The most widely used hybrid techniques also include MoM/GTD/FDFD/Eigen-Vector methods, MoM/PO, FEM/MoM, and MoM/MMP. Frequency and time-domain versions of several of these hybrid formalisms have been implemented in certain CEM codes which utilize FFT and inverse Fourier transform methods to relate time- and frequency-domain results. Recent work has been devoted to developing a fast finite element-boundary integral (FE-BI) technique.

C.17 Method of Moments (MoM) - This is a numerical technique based on the method of weighted residuals. It is synonymous with "surface integral technique" even though the method of weighted residuals can be applied to differential as well as integral equations. Moment method techniques apply thin-wire mesh/grid approximations or use perfectly electrically conducting (PEC) patch elements whose dimensions generally range between 0.1-0.25λ. It is most appropriate in analyzing electrically small-to-moderately sized unbounded radiation problems and excels at analyzing PEC configurations and homogeneous dielectrics. Wire segments can be loaded or defined as non-PEC type in certain MoM formulations. Electromagnetic fields are computed from wire mesh currents and patch surface current densities. In general, the method is not well suited to analyzing complex inhomogeneous geometries.

C.18 Multiple Multi-Pole (MMP) - The MMP is actually a code-based technique derived from the GMT method. It is a semi-analytic method for numerical field computations. Essentially, the field is expanded by a series of basis fields. Each of the basis fields is an analytic solution of the field equations within a homogeneous domain. The amplitudes of the basis fields are computed by a Generalized Point Matching Technique that is relatively efficient, accurate, and robust. Due to its close relations to analytic solutions, MMP is very useful and efficient when accurate and reliable solutions are desired.

C.19 Partial Element Equivalent Circuit Model (PEEC) - PEEC is based on the integral equation formulation. All structures to be modeled are divided into electrically small elements. An equivalent circuit describes the coupling between elements. Once the matrix of equivalent circuits is developed, then a circuit solver can be used to obtain a response for the system. It is mostly used for quasi-static partial inductance calculations to analyze printed circuit board electromagnetic radiation problems. One of the main advantages in using the PEEC method is the ability to add circuit elements into an EM simulator to model lumped circuit characteristics.

C.20 Pseudo-Spectral Time Domain Method (PSTD) - PSTD applies spectral domain techniques to a variety of electromagnetic boundary value problems. Using elementary concepts and methods can easily solve complex problems.

C.21 Shooting Bouncing Rays (SBR)/Physical Optics (PO)/Physical Theory of Diffraction (PTD) - SBR is a high-frequency method based on “shooting” large numbers of rays from transmitter locations against a given geometry consisting of “receivers”. The rays are shot without regard to receiver location (unlike the UTD or GTD method). These rays then “bounce off” reflecting planar surfaces (i.e., facets) following the laws of reflection. Since no particular ray paths are being searched to determine local minima, maxima or inflection the path for each ray is found relatively quickly. Many more rays can be considered than with GTD, but each SBR ray is much faster to compute since the shooting and bouncing process is very straightforward for a geometry consisting entirely of planar surfaces. Using the SBR method, one initially evaluates the field at a particular point, since one cannot be assured that all of the important SBR rays will pass through that point. One must use a “collection region” enclosing the point, and then use the rays that pass through this region to determine the field strength. The first-bounce PO plus the PTD contributions and the multi-bounce geometric optic ray contributions are included in the computation. These combined techniques work with planar, frequency selective surfaces (FSS) which are characterized by different types of impedance boundary conditions and complex material properties (i.e., non-PEC coatings, slabs or layers). Recent research has led to the development of non-planar or higher-order surface treatments.

C.22 Singularity Expansion Method (SEM) - Usual methods to treat time-dependent processes are either computation in the time domain by discretization with time steps or computation in the frequency domain and subsequent Fourier transform into the time domain. The SEM is a specific treatment of the time behavior via Laplace transform with complex frequencies.

Response waveforms of complicated scatterers in the resonance range are dominated by a few damped sinusoids. Such objects exhibit resonance frequencies (with singularities of the field) whose positions in the complex frequency plane are also determined by radiation losses. The problem is to determine the essential singularities which characterize a given object. So SEM is not so much an alternative method for solving field distributions but a procedure to be used in conjunction with other methods, in order to determine broadband responses.

C.23 Spectral Domain Approach (SDA) - The idea of the SDA is to utilize the possibilities of the Fourier transform for functions in space. A spectral domain can be assigned to one, two or three coordinates. The problem is solved in this spectral domain. The solution of the original problem results from an inverse transform of the solution in the spectral domain. In the example of a planar transmission line, the Fourier transform is applied in the direction parallel to the substrate and perpendicular to the strip. In the case of complex geometries the problem in the spectral domain has to be solved by semi-analytical or numerical methods, and the subsequent inverse transform has to be done numerically.

C.24 Thin-Wire Time Domain Method (TWTD)/Time Domain Moment Method (TDMM) - These are variations of the TLM and MoM thin-wire formulation in the time domain. The methods are based on the Integral Equation technique.

C.25 Transmission Line Method (TLM) - TLM belongs to the general class of differential time-domain numerical modeling methods. It is similar to the FDTD method in terms of its capabilities. Like FDTD, analysis is performed in the time domain and the entire region of the analysis is gridded. The basic TLM approach is to obtain a discrete model that is then solved exactly by numerical means. Approximations are only introduced at the discretization stage. For electromagnetic systems, the discrete model is formed by conceptually filling space with a network of transmission-lines in such a way that the voltage and current give information on the electric and magnetic fields. The point at which the transmission-lines intersect is referred to as a node and the most commonly used node for 3-D work is the symmetrical condensed node. Additional elements, such as transmission-line stubs, can be added to the node so that different material properties can be represented. Instead of interleaving E-field and H-field grids however, a single grid is established and the *nodes* of this grid are interconnected by virtual transmission lines. At each time step, voltage pulses are incident upon the node from each of the transmission lines. These pulses are then scattered to produce a new set of pulses that become incident on adjacent nodes at the next time step. Excitations at the source nodes continue to propagate to adjacent nodes through these transmission lines at each time step. These stubs are usually half the length of the mesh spacing and have characteristic impedance appropriate for the amount of loading desired. The advantages of using the TLM method are similar to those of the FDTD method. Complex, nonlinear materials are readily modeled. The disadvantages of the FDTD method are also shared by this technique. The primary disadvantage being that voluminous problems that must use a fine grid require excessive amounts of computation. Nevertheless, both the TLM and FDTD techniques are very powerful and widely used.

C.26 Vector Parabolic Equation Technique (VPE) - VPE methods, used to analyze radio wave propagation in radar and radio communication systems, are new and powerful techniques that have become the dominant tool for assessing clear-air and terrain effects on propagation. The technique is key to engineers and researchers analyzing diffraction and ducting in radio communication systems.