Time-Domain Extraction and SPICE Macromodeling

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Previous Presentations

- "Time-Domain Macromodel Extraction" (briefly) www.ibis.org/summits/may21/ross1.pdf
  - Nomenclature and some mathematical identities
  - Duality (see presentation for details)
  - Key equation for steepest descent optimization
  - Laplace transform optimization flow for simultaneous numerator and denominator coefficients
  - Last column mathematics for a companion matrices (not shown here)
  - In place algorithms (mentioned)
  - Constraints

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Previous Presentations (continued)

- "SPICE Macromodel Generation" www.ibis.org/summits/may21/ross2.pdf
  - Mostly included here
  - Main point is to illustrate a set of circuits (and their efficiencies) for generating poles and zeros
  - Pole/zero cancellation is illustrated
  - Cascaded circuits were common practice in operational amplifier data book/sheet models

TIME-DOMAIN EXTRACTION

- Goal – Low-order Laplace transform network function from time-domain measurements (or simulations) as a ratio of polynomials in s
  - Noisy measurements
  - Uncoupled networks
  - Least squared error steepest descent algorithm
- Show some not so well-known mathematical identities
  - Duality
  - Last column mathematics for functions of companion matrices
  - In place algorithms
- Based on original correspondence 1969 – 1972 with Janez Valand (Yugoslavia/Croatia) and actual product implementation (1990's)
- Derivations and proofs not shown
  - Proofs based on power series expansions and companion matrix relationships

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Special Notation - Equations

Laplace Transform	$X(s)=\dfrac{a_{n-1}s^{n-1}+\cdots+a_0}{s^n+b_{n-1}s^{n-1}+\cdots+b_0},$
Differential Equation	$x^n(t)+b_{n-1}x^{n-1}(t)+\cdots+b_0x(t)=0$ initial conditions, $x(0),\cdots,x^{n-1}(0),$
Difference Equation	$x_n(t)+d_{n-1}x_{n-1}(t)+\cdots+d_0x_0(t)=0$ initial conditions, $x_0(0),\cdots,x_{n-1}(0),$
Z Transform	$Z(z)=\dfrac{z(c_{n-1}z^{n-1}+\cdots+c_0)}{z^n+d_{n-1}z^{n-1}+\cdots+d_0}.$

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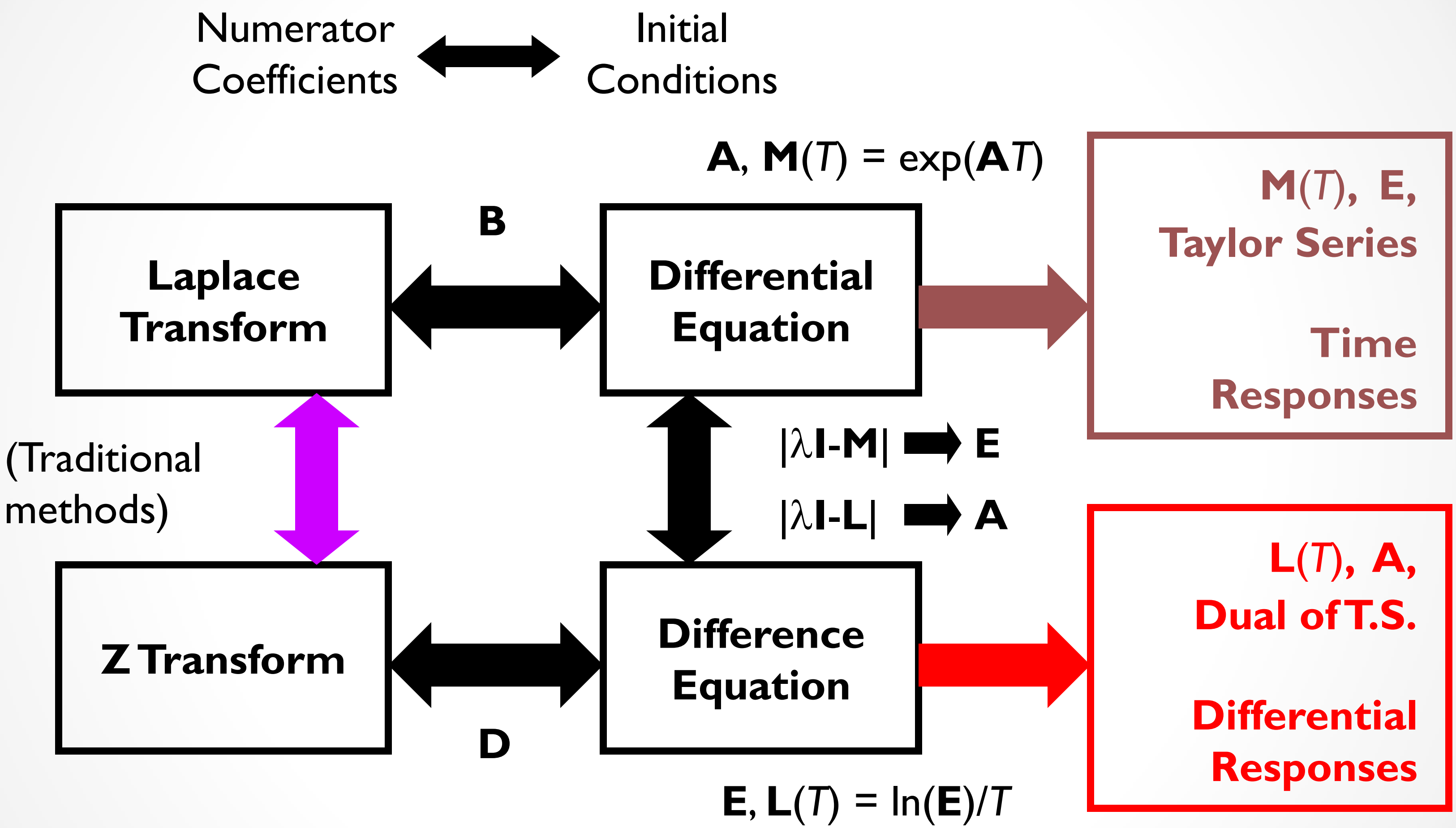
Conversions and Responses



Differential Equations

$d\mathbf{x}(t)/dt = \mathbf{A}\mathbf{x}(t)$

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \\ -b_0 & -b_1 & \cdots & -b_{n-1} \end{bmatrix}$$

$\mathbf{x}(t+T) = \mathbf{M}\mathbf{x}(t)$

$\mathbf{M} = \exp(\mathbf{A}T)$

Difference Equations

$\mathbf{z}(t+T) = \mathbf{E}\mathbf{z}(t)$

$$\mathbf{E} = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 1 \\ -d_0 & -d_1 & \cdots & -d_{n-1} \end{bmatrix}$$

$d\mathbf{z}(t)/dt = \mathbf{L}\mathbf{z}(t)$

$\mathbf{E} = \exp(\mathbf{L}T)$

$\mathbf{L} = \ln(\mathbf{E})/T$

Companion Matrices

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Differential Equations

$\mathbf{x}(t) = [x^0(t), x^1(t), \cdots, x^{n-1}(t)]^T,$

$\mathbf{a} = [a_{n-1}, \cdots, a_0]^T,$

$$\mathbf{B} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ b_{n-1} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ b_2 & b_3 & \cdots & 1 & 0 \\ b_1 & b_2 & \cdots & b_{n-1} & 1 \end{bmatrix}$$

$\mathbf{a} = \mathbf{B}\mathbf{x}(0)$

Difference Equations

$\mathbf{z}(t) = [x_0(t), x_1(t), \cdots, x_{n-1}(t)]^T$

$\mathbf{c} = [c_{n-1} \cdots c_0]^T$

$$\mathbf{D} = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ d_{n-1} & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ d_2 & d_3 & \cdots & 1 & 0 \\ d_1 & d_2 & \cdots & d_{n-1} & 1 \end{bmatrix}$$

$\mathbf{c} = \mathbf{D}\mathbf{z}(0)$

Recursive Taylor Series (Repeat b and c)

a) Initialize: $i = 1, \ldots, n-1$

$$x(0) = a_{n-1} \qquad x^i(0) = a_{n-1-i} - \sum_{j=0}^{i-1} b_{n-i-j}\, x^j(0)$$

b) Extend: $i = n, \ldots, p$

$$x^i(t) = -\sum_{j=0}^{n-1} b_j\, x^{i-n-j}(t)$$

c) Next time step: $i = 0, \ldots, n-1$ (Taylor series)

$$x^i(t+T) = \sum_{j=i}^{p} x^j(t) \frac{T^{j-i}}{(j-i)!}$$

R. I. Ross, "Evaluating the Transient Response of a Network Function," *Proc. IEEE*, vol.55, pp. 615-616, May 1967

Differential Eq'n Sensitivities

$$\left[\frac{\partial x(t)}{\partial a_0}, \frac{\partial x(t)}{\partial a_1}, \dots, \frac{\partial x(t)}{\partial a_{n-1}}\right]^T =$$

$$\left[\frac{\partial x(t)}{\partial a_0}, \frac{\partial x^1(t)}{\partial a_0}, \dots, \frac{\partial x^{n-1}(t)}{\partial a_0}\right]^T$$

$$\left[\frac{\partial x(t)}{\partial b_0}, \frac{\partial x(t)}{\partial b_1}, \dots, \frac{\partial x(t)}{\partial b_{n-1}}\right]^T =$$

$$\left[\frac{\partial x(t)}{\partial b_0}, \frac{\partial x^1(t)}{\partial b_0}, \dots, \frac{\partial x^{n-1}(t)}{\partial b_0}\right]^T$$

$$\frac{\partial x^i(t)}{\partial a_j \partial b_k} = \frac{\partial x^{i+j+k}(t)}{\partial a_0 \partial b_0}$$

Difference Eq'n Sensitivities

$$\left[\frac{\partial x(t)}{\partial c_0}, \frac{\partial x(t)}{\partial c_1}, \dots, \frac{\partial x(t)}{\partial c_{n-1}}\right]^T =$$

$$\left[\frac{\partial x(t)}{\partial c_0}, \frac{\partial x_1(t)}{\partial c_0}, \dots, \frac{\partial x_{n-1}(t)}{\partial c_0}\right]^T$$

$$\left[\frac{\partial x(t)}{\partial d_0}, \frac{\partial x(t)}{\partial d_1}, \dots, \frac{\partial x(t)}{\partial d_{n-1}}\right]^T =$$

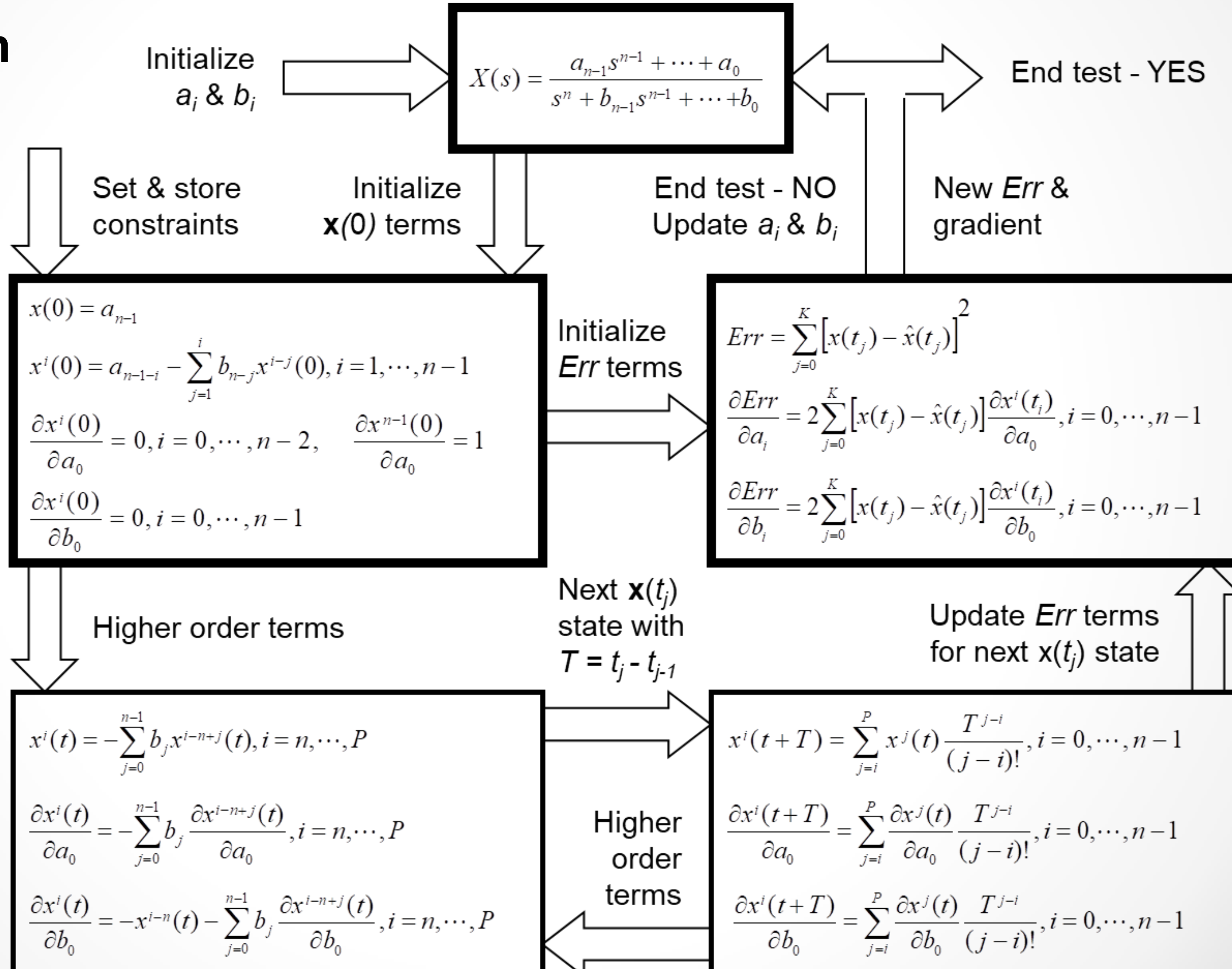
$$\left[\frac{\partial x(t)}{\partial d_0}, \frac{\partial x_1(t)}{\partial d_0}, \dots, \frac{\partial x_{n-1}(t)}{\partial d_0}\right]^T$$

$$x_j(t) = x(t + jT), \quad j = 0, \dots, n-1$$

$$\frac{\partial x_i(t)}{\partial c_j \partial d_k} = \frac{\partial x_{i+j+k}(t)}{\partial c_0 \partial d_0}$$

Laplace Transform Extraction (T.S.)

Expanded on next slide



Initialize a_i & b_i

$$X(s) = \frac{a_{n-1}s^{n-1} + \cdots + a_0}{s^n + b_{n-1}s^{n-1} + \cdots + b_0}$$

End test - YES

Set & store constraints

Initialize **x**(0) terms

End test - NO
Update a_i & b_i

New *Err* & gradient

$$x(0) = a_{n-1}$$

$$x^i(0) = a_{n-1-i} - \sum_{j=1}^{i} b_{n-j} x^{i-j}(0), i = 1, \cdots, n-1$$

$$\frac{\partial x^i(0)}{\partial a_0} = 0, i = 0, \cdots, n-2, \qquad \frac{\partial x^{n-1}(0)}{\partial a_0} = 1$$

$$\frac{\partial x^i(0)}{\partial b_0} = 0, i = 0, \cdots, n-1$$

Initialize *Err* terms

$$Err = \sum_{j=0}^{K} \left[x(t_j) - \hat{x}(t_j)\right]^2$$

$$\frac{\partial Err}{\partial a_i} = 2\sum_{j=0}^{K} \left[x(t_j) - \hat{x}(t_j)\right] \frac{\partial x^i(t_i)}{\partial a_0}, i = 0, \cdots, n-1$$

$$\frac{\partial Err}{\partial b_i} = 2\sum_{j=0}^{K} \left[x(t_j) - \hat{x}(t_j)\right] \frac{\partial x^i(t_i)}{\partial b_0}, i = 0, \cdots, n-1$$

Higher order terms

Next **x**(t_j) state with $T = t_j - t_{j-1}$

Update *Err* terms for next x(t_j) state

$$x^i(t) = -\sum_{j=0}^{n-1} b_j x^{i-n+j}(t), i = n, \cdots, P$$

$$\frac{\partial x^i(t)}{\partial a_0} = -\sum_{j=0}^{n-1} b_j \frac{\partial x^{i-n+j}(t)}{\partial a_0}, i = n, \cdots, P$$

$$\frac{\partial x^i(t)}{\partial b_0} = -x^{i-n}(t) - \sum_{j=0}^{n-1} b_j \frac{\partial x^{i-n+j}(t)}{\partial b_0}, i = n, \cdots, P$$

Higher order terms

$$x^i(t+T) = \sum_{j=i}^{P} x^j(t) \frac{T^{j-i}}{(j-i)!}, i = 0, \cdots, n-1$$

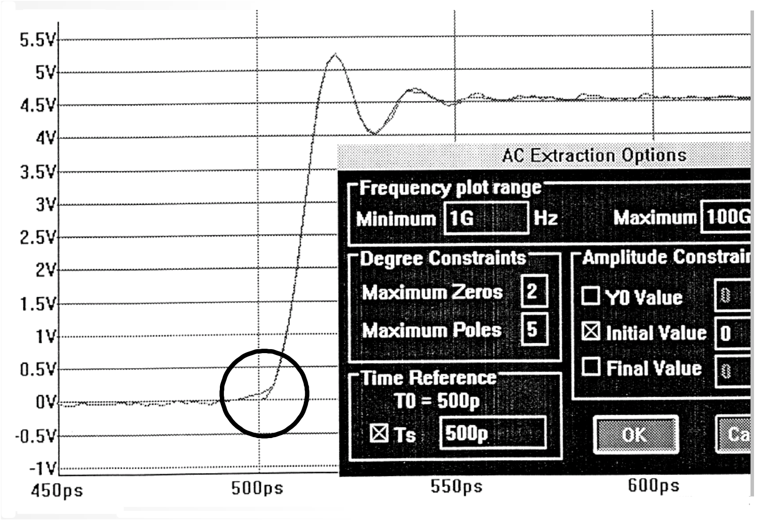
$$\frac{\partial x^i(t+T)}{\partial a_0} = \sum_{j=i}^{P} \frac{\partial x^j(t)}{\partial a_0} \frac{T^{j-i}}{(j-i)!}, i = 0, \cdots, n-1$$

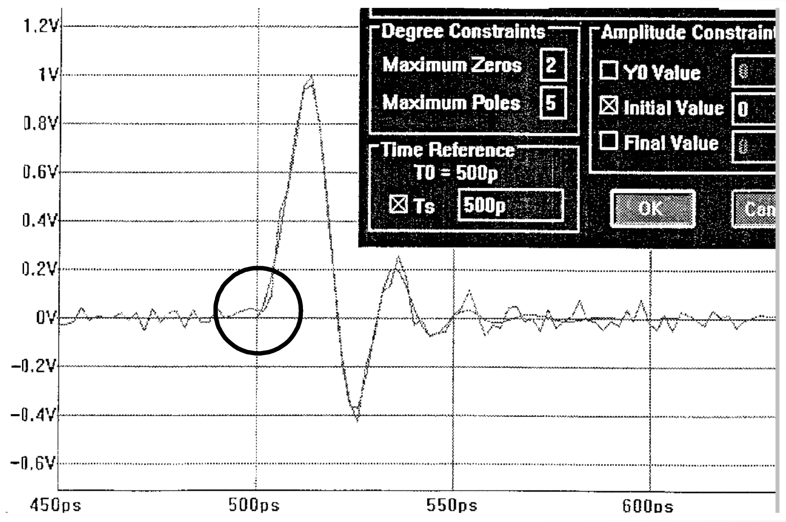
$$\frac{\partial x^i(t+T)}{\partial b_0} = \sum_{j=i}^{P} \frac{\partial x^j(t)}{\partial b_0} \frac{T^{j-i}}{(j-i)!}, i = 0, \cdots, n-1$$

Available Constraints

- Denominator degree (number of poles)
- Ideal step response (b_0=0.0)
- Numerator degree (number of zeros, where lower degree produces less leading edge ripple)
- Set t_S start value, DC offset (y_0)
- Minimum and maximum frequencies for log frequency domain plots
- Initial value
- Final value

Constraints (Oscilloscope Step and Impulse Response Extractions)





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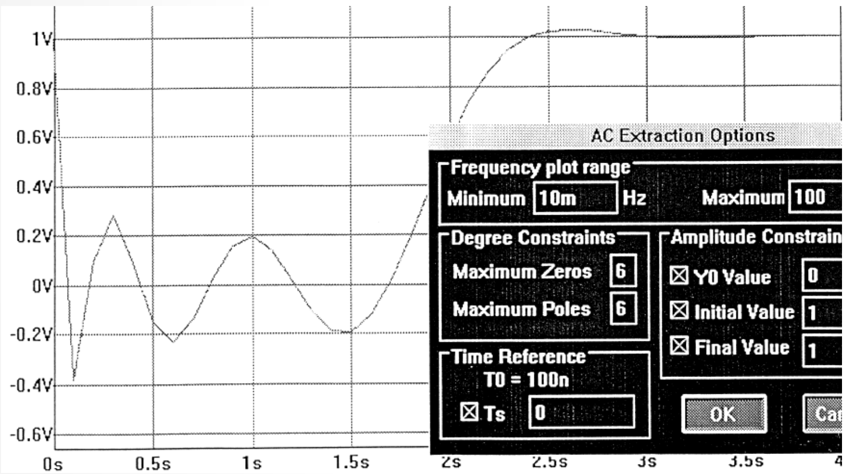
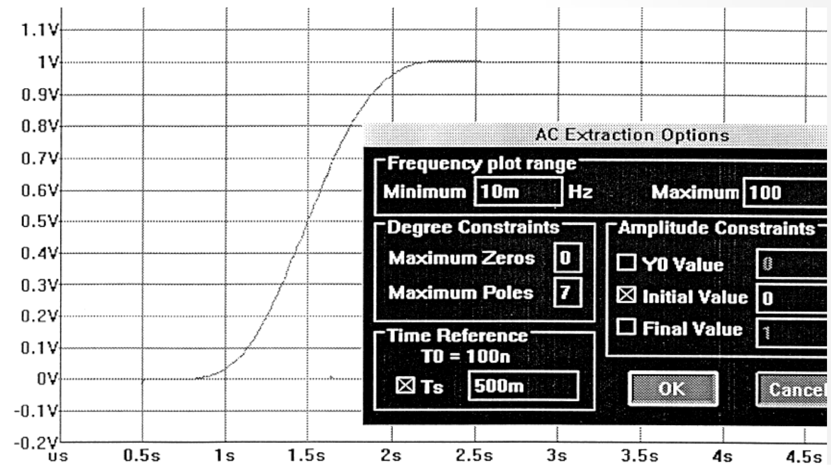
Constraints – Maximally Flat Envelope Delay Networks





Conclusions

- Early (1970's) algorithm outlined for physical measurement-based time-domain extraction
- Most calculations done in place to save memory
- Most calculations used last-column matrix mathematics
- Many subroutines worked in both the difference equation and differential equation domains
- Laplace transform formulation allowed practical constraints to be implemented
- Result was a Laplace transform polynomial ratio, N(*s*)/D(*s*), from a time-domain response

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GENERATING SPICE MACROMODELS

- Preliminary material
- Macromodel references
- Networks for poles and zeros (and their efficiencies)
- Operational amplifier open-loop response
- Operational amplifier macromodel example
- Conclusion

Automatic Implementation

- **Starting point – Laplace transform H(*s*) = N(*s*)/D(*s*) as ratio of polynomials is *s***
- **Laplace transform, pole/zero, or pole/residue formats not interchangeable between EDA tools**
- **Lowest common denominator – Berkeley SPICE RLC elements and controlled gain elements**
- **Implementation based on solving for poles and zeros and then cascading unit gain stages with efficient grouping.**
- **Automatic node numbering**
- **Stages referenced to one megohm (MΩ) resistor**

SPICE Macromodels

- G. Boyle, B. Cohn, D. Pederson, J. Solomon, “Macromodeling of Integrated Circuit Operational Amplifiers”, IEEE Journal of Solid-State Circuits, Vol. SC-9, No. 6, Dec. 1974, pp. 353-363
  - Dominant and second real pole
  - A commercial vendor macromodel adapted to illustrate a general behavioral macromodel generation strategy
  - Strategy can be applied to high-speed networks
- Cascaded SPICE elements are common practice from several vendors, but some macromodels use:
  - Real left-hand plane (LHP) poles and zeros
  - No right-hand plane (RHP) zeros
  - No complex poles or zeros
  - Extractions often based on frequency domain magnitude and phase measurements

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Networks

- **Basic** Stages (simple poles and zeros or combinations)
- **Constructed** Stages (combining several basic networks for an overall set of poles and zeros
- **Utility** Networks for pole/zero cancelation
- **All-pass** Networks for cancellations
- Efficiencies relative to a single-pole stage (combined P+Z stages usually more efficient)
  - Parts per pole+zero relative to **3.0**
  - Nodes per pole+zero relative to **1.0**

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Basic Stages

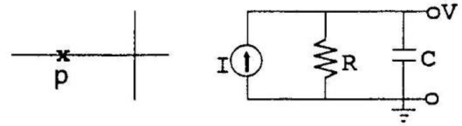
BASIC STAGES:

Real Zeros	Cmplx Conj Zeros	Real Poles	Cmplx Conj Poles	Poles+ Zeros	Stages	Parts	New Nodes	Parts Per P + Z	Nodes Per P + Z
1. Real Pole									
		X		1	1	3	1	**3**	**1**
2. Real Pole, Real Zero 2a. z < p			2b. z > p						
O		X		2	a, b: 1	4	2	**2**	**1**
3. Complex-Conjugate Poles									
			X X	2	1	4	2	**2**	**1**
4. Complex-Conjugate Poles, Real Zero 4a. $z < 2\sigma$			4b. $z > (\omega^2+\sigma^2) / 2\sigma$						
O			X X	3	a: 1 b: 1	5 5	2 3	**1.67** **1.67**	**0.67** **1**

Basic Stages

BASIC STAGES:

1. Real Pole

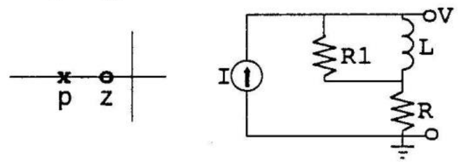


$$\frac{V}{I} = \frac{R}{(1 + s/p)}$$

$C = 1 / R p$

2. Real Pole, Real Zero

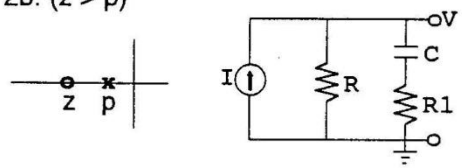
2a. (z < p)



$$\frac{V}{I} = \frac{R(1 + s/z)}{(1 + s/p)}$$

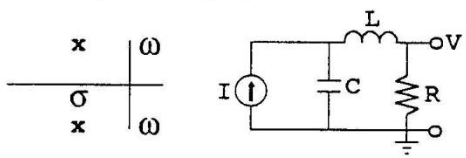
$R1 = R / (p / z - 1)$
$L = R1 / p$

2b. (z > p)



$R1 = R / (z / p - 1)$
$C = 1 / R1\, z$

3. Complex-Conjugate Poles



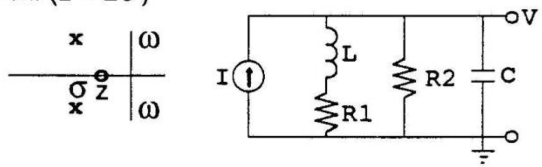
$$\frac{V}{I} = \frac{R}{(1 + 2\sigma s / (\omega^2+\sigma^2) + s^2 / (\omega^2+\sigma^2))}$$

$L = R / 2\sigma$
$C = 1 / L (\omega^2+\sigma^2)$

Basic Stages (Continued)

4. Complex-Conjugate Poles, Real Zero

4a. ($z < 2\sigma$)



$$\frac{V}{I} = \frac{R\,(1 + s/z)}{(1 + 2\sigma s/(\omega^2+\sigma^2) + s^2/(\omega^2+\sigma^2))}$$

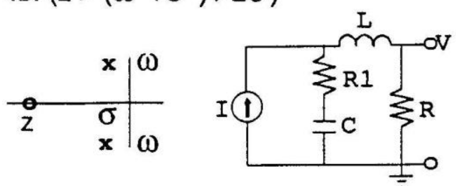
$C = z / R\,(\omega^2+\sigma^2)$

$R2 = 1 / C\,(2\sigma - z)$

$R1 = R\,R2 / (R2 - R)$

$L = R1 / z$

4b. ($z > (\omega^2+\sigma^2) / 2\sigma$)



$C = (2\sigma / (\omega^2+\sigma^2) - 1 / z) / R$

$R1 = 1 / C\,z$

$L = 1 / C\,(\omega^2+\sigma^2)$

Constructed Stages

CONSTRUCTED STAGES:

Real Zeros	Cmplx Conj Zeros	Real Poles	Cmplx Conj Poles	Poles+ Zeros	Stages	Parts	New Nodes	Parts Per P + Z	Nodes Per P + Z
4. **Complex-Conjugate Poles, Real Zero** (10 & 2) 4c. $(\omega^2+\sigma^2) / 2\sigma > z > 2\sigma$									
O			X X	3	c: 2	8	4	**2.67**	**1.33**
5. **Complex-Conjugate Poles, Two Real Zeros** (11 & 2)									
O O			X X	4	2 max: 3	9 13	5 7	**2.25** **3.25**	**1.25** **1.75**
6. **Complex-Conjugate Poles, Complex-Conjugate Zeros** (11, 12 & 2)									
	O O		X X	4	3	14	7	**3.5**	**1.75**
7. **Two Real Poles, Complex-Conjugate Zeros** (12 & 2)									
	O O	X X		4	2 max: 3	9 13	4 6	**2.25** **3.25**	**1** **1.5**
8. **Complex-Conjugate Poles, Real Pole, Complex-Conjugate Zeros** (12 & 4)									
	O O	X	X X	5	min: 2 2 max: 4	10 10 17	5 6 8	**2** **2** **3.4**	**1** **1.2** **1.6**
9. **Complex-Conjugate Poles Real Pole, Complex-Conjugate Zeros, Real Zero** (11, 12 & 2)									
O	O O	X	X X	6	3 max: 5	14 22	7 11	**2.33** **3.67**	**1.14** **1.83**

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Utility Networks and Combinations for Construction

UTILITY NETWORKS AND COMBINATIONS FOR CONSTRUCTION:

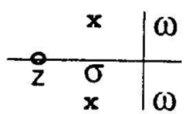
Real Zeros	Cmplx Conj Zeros	Real Poles	Cmplx Conj Poles	Poles+ Zeros	Stages	Parts	New Nodes	Parts Per P + Z	Nodes Per P + Z
10. Complex-Conjugate Poles, Fixed Real Zero									
O ^ fixed			X X	3	1	4	2	1.33	0.67
11. Complex-Conjugate Poles, Real Zero, Fixed Real Zero									
11a. $z < 2\sigma$	11b. $z > (\omega^2+\sigma^2) / 2\sigma$	11c. $(\omega^2+\sigma^2) / 2\sigma > z > 2\sigma$ (by combination of 11a & 2)							
O O ^or^ fix			X X	4	a: 1 b: 1 c: 2	5 5 9	3 3 5	1.25 1.25 2.25	0.75 0.75 1.25
12. Complex-Conjugate Zeros, Real Pole, Fixed Real Pole									
12a. $p < 2\sigma$	12b. $p > (\omega^2+\sigma^2) / 2\sigma$	12c. $(\omega^2+\sigma^2) / 2\sigma > p > 2\sigma$ (by combination of 12a & 2)							
	O O	X X ^or^ fix		4	a: 1 b: 1 c: 2	5 5 9	2 2 4	1.25 1.25 2.25	0.5 0.5 1

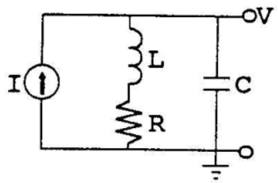
Utility Networks for Construction

UTILITY NETWORKS FOR CONSTRUCTION:

10. Complex-Conjugate Poles, Fixed Real Zero

$$\frac{V}{I} = \frac{R\,(1 + s/z)}{(1 + 2\sigma s/(\omega^2+\sigma^2) + s^2/(\omega^2+\sigma^2))}$$





$L = R / 2\sigma$

$C = 1 / L\,(\omega^2+\sigma^2)$

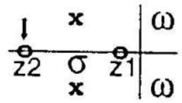
$z = 2\sigma$

Utility Networks (Continued)

11. Complex-Conjugate Poles, Real Zero, Fixed Real Zero

$$\frac{V}{I} = \frac{R\,(1 + s/z1)\,(1 + s/z2)}{(1 + 2\sigma\, s / (\omega^2+\sigma^2) + s^2 / (\omega^2+\sigma^2))}$$

11a. $(z1 < 2\sigma)$

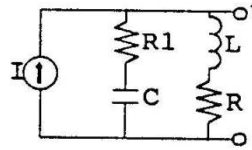


$L = R / z1$

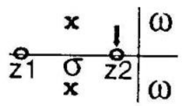
$C = 1 / L\,(\omega^2+\sigma^2)$

$R1 = R / (2\sigma / z1 - 1)$

$z2 = 1 / R1\,C$



11b. $(z1 > (\omega^2+\sigma^2) / 2\sigma)$



$C = (2\sigma / (\omega^2+\sigma^2) - 1 / z1) / R$

$R1 = 1 / C\,z1$

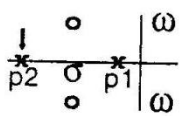
$L = 1 / C\,(\omega^2+\sigma^2)$

$z2 = R / L$

12. Complex-Conjugate Zeros, Real Pole

$$\frac{V}{I} = \frac{R\,(1 + 2\sigma\, s / (\omega^2+\sigma^2) + s^2 / (\omega^2+\sigma^2))}{(1 + s/p1)\,(1 + s/p2)}$$

12a. $(p1 < 2\sigma)$

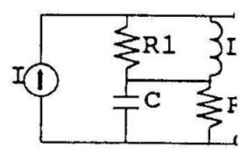


$C = 1 / R\,p1$

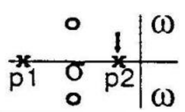
$L = 1 / C\,(\omega^2+\sigma^2)$

$R1 = R / (2\sigma / p1 - 1)$

$p2 = R1 / L$



12b. $(p1 > (\omega^2+\sigma^2) / 2\sigma)$



$C = 1 / L\,(\omega^2+\sigma^2)$

$L = R\,(2\sigma / (\omega^2+\sigma^2) - 1 / p1)$

$R1 = L / p1$

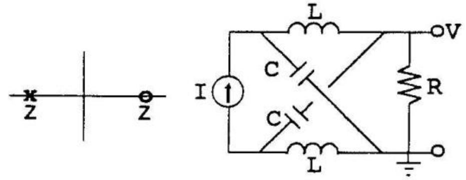
$p2 = 1/RC$

All-Pass Networks (Mirrored P/Z)

ALL-PASS NETWORKS FOR RIGHT-HAND PLANE ZEROS (WITH MIRRORED POLES):

13. RHP Real Zero, Fixed Real Pole

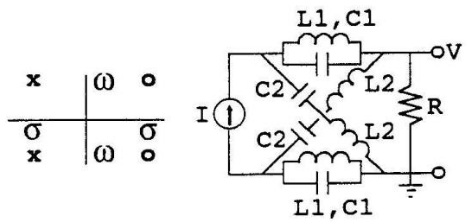
$$\frac{V}{I} = \frac{R\,(1 - s / z)}{(1 + s / z)}$$



$L = R / z$
$C = 1 / R^2$

14. RHP Complex-Conjugate Zeros, Fixed Complex-Conjugate Poles

$$\frac{V}{I} = \frac{R\,(1 - 2\sigma\, s / (\omega^2+\sigma^2) + s^2 / (\omega^2+\sigma^2))}{(1 + 2\sigma\, s / (\omega^2+\sigma^2) + s^2 / (\omega^2+\sigma^2))}$$

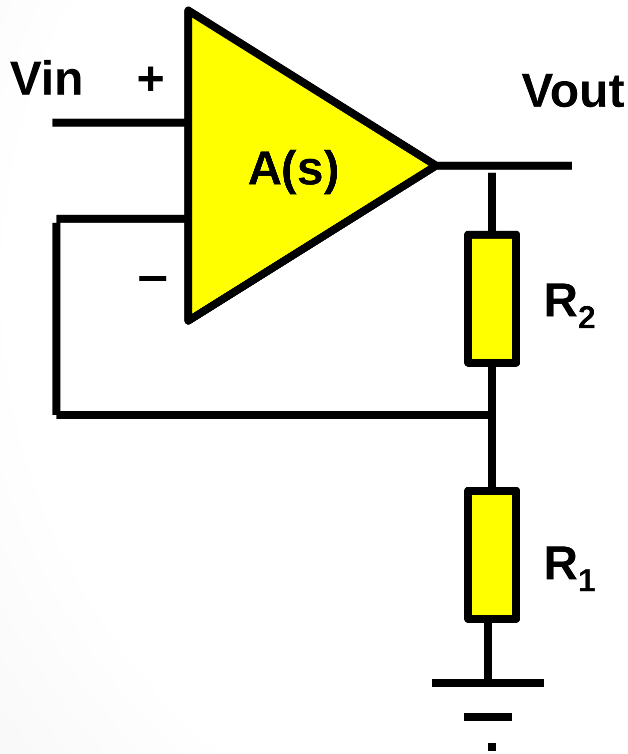


$C1 = 1 / 2R\,\sigma$
$L1 = 1 / C1\,(\omega^2+\sigma^2)$
$L2 = R^2\,C1$
$C2 = 1 / L2\,(\omega^2+\sigma^2)$

ALL-PASS NETWORKS FOR RIGHT HAND PLANE ZEROS (WITH MIRRORED POLES):

Real Zeros	Cmplx Conj Zeros	Real Poles	Cmplx Conj Poles	Poles+ Zeros	Stages	Parts	New Nodes	Parts Per P + Z	Nodes Per P + Z
13. RHP Real Zero, Fixed Real Pole									
RHP **O**		**X** ^ fixed		2	1	6	3	**3**	**1.5**
14. RHP Complex-Conjugate Zeros, Fixed Complex-Conjugate Poles									
	RHP **O** RHP **O**		**X** fixed **X** fixed	4	1	10	5	**2.5**	**1.25**

Open Loop (OL) AC Model from Closed Loop (CL) Response



Extracted H(s) = Vout/Vin from time response = N(s)/D(s)

$G = (R_1 + R_2)/R_1$

$H(s) = A(s)/[1 + A(s)/G]$

$A(s) = H(s) + H(s)A(s)/G$

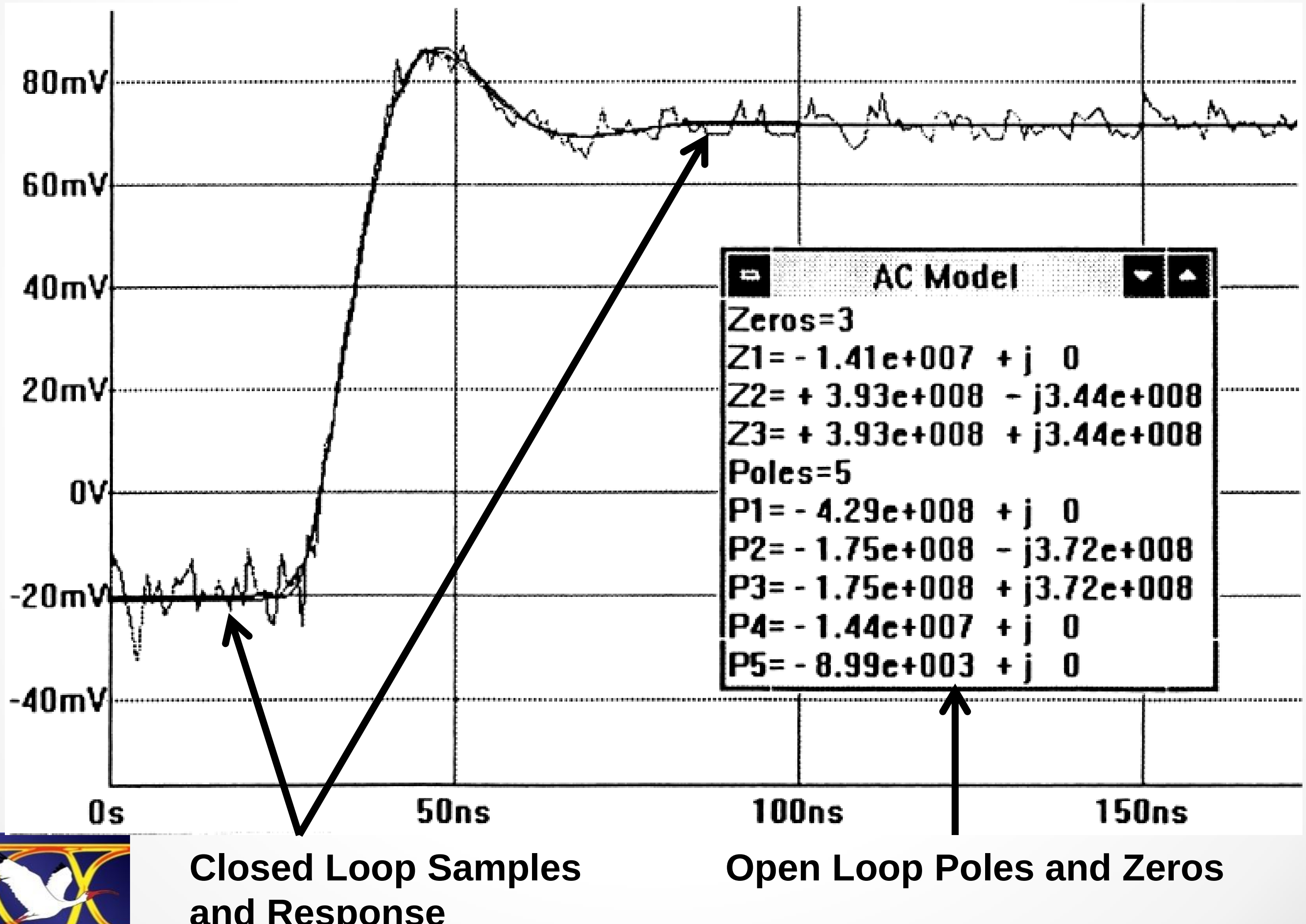
$A(s) = H(s)/[1 - H(s)/G]$

$A(s) = [N(s)/D(s)]/[1 - N(s)/GD(s)]$

$A(s) = N(s)/[D(s) - N(s)/G]$

Poles and zeros of A(s) produces OL AC Model

Operational Amplifier AC Model Example



Generated SPICE Macromodel

AC Model

```
Zeros=3
Z1= - 1.41e+007  + j  0
Z2= + 3.93e+008  - j3.44e+008
Z3= + 3.93e+008  + j3.44e+008
Poles=5
P1= - 4.29e+008  + j  0
P2= - 1.75e+008  - j3.72e+008
P3= - 1.75e+008  + j3.72e+008
P4= - 1.44e+007  + j  0
P5= - 8.99e+003  + j  0
```

```
*
* SETUP PARAMETERS
*  NPN Bipolar Junction Transistor Input
*  Vcc = 15 V, Vee = -15 V
*  Input Stage Tail Current = 0.1 mA
*
* MEASURED (OR USER OVERRIDDEN) PARAMETERS
*
*  Srp = 135.3 V/us, Srn = 135.3 V/us
*  Avd = 94.211 dB at RL(Load) = 1e+009 kOhms
*  f(0dB) = 71.8 MHz, Phi(Phase Margin) = 222.2 deg
*
* ZEROS        Radians                     MHz
*  Real            Imaginary         Real              Imaginary
*  -1.41e+007      0                 -2.24408          0
*  3.93e+008       -3.44e+008        62.5479           -54.7493
*  3.93e+008       3.44e+008         62.5479           54.7493
*
* POLES        Radians                     MHz
*  Real            Imaginary         Real              Imaginary
*  -8990           0                 -0.0014308        0
*  -1.44e+007      0                 -2.29183          0
*  -1.75e+008      -3.72e+008        -27.8521          -59.2056
*  -1.75e+008      3.72e+008         -27.8521          59.2056
*  -4.29e+008      0                 -68.2775          0
*
*               NON-INVERTING INPUT
*               |  INVERTING INPUT
*               |  |  POSITIVE POWER SUPPLY
*               |  |  |  NEGATIVE POWER SUPPLY
*               |  |  |  |  OUTPUT
*               |  |  |  |  |
.SUBCKT OPamp   1  2 99 50 52
*
* INPUT STAGE (NO POLE)
*
R1    2    3    5e+011
R2    1    3    5e+011
R3    5    99   2930.88
R4    6    99   2930.88
I1    4    50   0.0001
IOS   1    2    1.095e-009
EOS   9    1    POLY(1)    59    49    -0.001136 1
Q1    5    2    7  QX
Q2    6    9    8  QX
R5    4    7    2421.07
R6    4    8    2421.07
*
EREF  98   0    49    0    1
*
```

Poles and zeros shown in radians and MHz in the macromodel

Teraspeed Labs

Low Frequency Pole

```
EREF   98    0     49    0     1
*
* SECOND STAGE POLE AT 1430.8 HZ
*
R21    100   98    1.50501e+008
C21    100   98    7.39098e-013
G21    100   98    6     5     0.000341195
V21    99    21    2.39393
V22    22    50    2.41393
D21    100   21    DX
D22    22    100   DX
*
* RHP COMPLEX ZEROS AT 62.5479 +/- j 54.7493 MHZ
* NEW LHP COMPLEX POLES AT 62.5479 +/- j 54.7493 MHZ
*
R101   105   98    1e+006
C101   101   98    1.27226e-015
C102   105   102   1.27226e-015
C103   105   103   2.88139e-015
C104   104   98    2.88139e-015
L101   101   98    0.00288139
L102   105   102   0.00288139
L103   101   103   0.00127226
L104   102   104   0.00127226
G101   102   101   49    100   1e-006
*
```

MHz

Real	Imaginary
-2.24408	0
62.5479	-54.7493
62.5479	54.7493

MHz

Real	Imaginary
-0.0014308	0
-2.29183	0
-27.8521	-59.2056
-27.8521	59.2056
-68.2775	0

```
*************** CONSTRUCTED STAGE ****************
*
* LHP REAL ZERO AT 2.24408 MHZ
* LHP COMPLEX ZEROS AT 62.5479 +/- j 54.7493 MHZ
* LHP REAL POLE AT 2.29183 MHZ
* LHP COMPLEX POLES AT 27.8521 +/- j 59.2056 MHZ
*
* LHP ZERO AT 2.24408 MHZ
* NEW LHP ZERO AT 80.0792 MHZ
* LHP COMPLEX POLES AT 27.8521 +/- j 59.2056 MHZ
*
R106   108   106   1e+006
R107   108   107   2.38227e+007
C106   107   98    8.34275e-017
L106   106   98    0.070922
G106   108   98    49    105   1e-006
*
* LHP COMPLEX ZEROS AT 62.5479 +/- j 54.7493 MHZ
* LHP POLE AT 2.29183 MHZ
* NEW LHP POLE AT 56.2663 MHZ
*
R109   110   109   1e+006
R110   109   98    18662.5
C109   110   109   6.94444e-014
L109   109   98    5.27888e-005
G109   110   98    49    108   1e-006
*
* LHP ZERO AT 56.2663 MHZ
* LHP POLE AT 80.0792 MHZ
*
R111   112   111   1e+006
R112   111   98    423218
L111   111   98    0.000841133
G111   112   98    49    110   1e-006
*
************ END OF CONSTRUCTED STAGE ************
```

Teraspeed Labs

3 Poles, 3 Zeros using Cancellations

```
EREF   98   0   49   0   1
*
* SECOND STAGE POLE AT 1430.8 HZ
*
R21   100   98   1.50501e+008
C21   100   98   7.39098e-013
G21   100   98   6   5   0.000341195
V21   99    21   2.39393
V22   22    50   2.41393
D21   100   21   DX
D22   22    100  DX
*
* RHP COMPLEX ZEROS AT 62.5479 +/- j 54.7493 MHZ
* NEW LHP COMPLEX POLES AT 62.5479 +/- j 54.7493 MHZ
*
R101  105   98   1e+006
C101  101   98   1.27226e-015
C102  105   102  1.27226e-015
C103  105   103  2.88139e-015
C104  104   98   2.88139e-015
L101  101   98   0.00288139
L102  105   102  0.00288139
L103  101   103  0.00127226
L104  102   104  0.00127226
G101  102   101  49   100   1e-006
*
```

Cancellation

(9)

(14)

```
*************** CONSTRUCTED STAGE ****************
*
* LHP REAL ZERO AT 2.24408 MHZ
* LHP COMPLEX ZEROS AT 62.5479 +/- j 54.7493 MHZ
* LHP REAL POLE AT 2.29183 MHZ
* LHP COMPLEX POLES AT 27.8521 +/- j 59.2056 MHZ
*
* LHP ZERO AT 2.24408 MHZ
* NEW LHP ZERO AT 80.0792 MHZ
* LHP COMPLEX POLES AT 27.8521 +/- j 59.2056 MHZ
*
R106  108   106  1e+006
R107  108   107  2.38227e+007
C106  107   98   8.34275e-017
L106  106   98   0.070922
G106  108   98   49   105   1e-006
*
* LHP COMPLEX ZEROS AT 62.5479 +/- j 54.7493 MHZ
* LHP POLE AT 2.29183 MHZ
* NEW LHP POLE AT 56.2663 MHZ
*
R109  110   109  1e+006
R110  109   98   18662.5
C109  110   109  6.94444e-014
L109  109   98   5.27888e-005
G109  110   98   49   108   1e-006
*
* LHP ZERO AT 56.2663 MHZ
* LHP POLE AT 80.0792 MHZ
*
R111  112   111  1e+006
R112  111   98   423218
L111  111   98   0.000841133
G111  112   98   49   110   1e-006
*
************ END OF CONSTRUCTED STAGE ************
```

(11)

(12)

(2)

Cancellations

MHz

Real	Imaginary
-2.24408	0
62.5479	-54.7493
62.5479	54.7493

MHz

Real	Imaginary
-0.0014308	0
-2.29183	0
-27.8521	-59.2056
-27.8521	59.2056
-68.2775	0

Teraspeed Labs

Last Pole and Last Stages

```
                 MHz
Real                 Imaginary
-2.24408             0
62.5479              -54.7493
62.5479              54.7493

                 MHz
Real                 Imaginary
-0.0014308           0
-2.29183             0
-27.8521             -59.2056
-27.8521             59.2056
-68.2775             0
```

```
*
* LHP POLE AT 68.2775 MHZ
*
R113    113    98    1e+006
C113    113    98    2.331e-015
G113    113    98    49    112    1e-006
*
* COMMON MODE GAIN STAGE WITH ZERO AT 20 KHZ
*
R57     59     57    1e+006
C57     59     57    7.95775e-012
R58     59     98    1
E57     57     98    49    3      3.55745
*
```

(1)

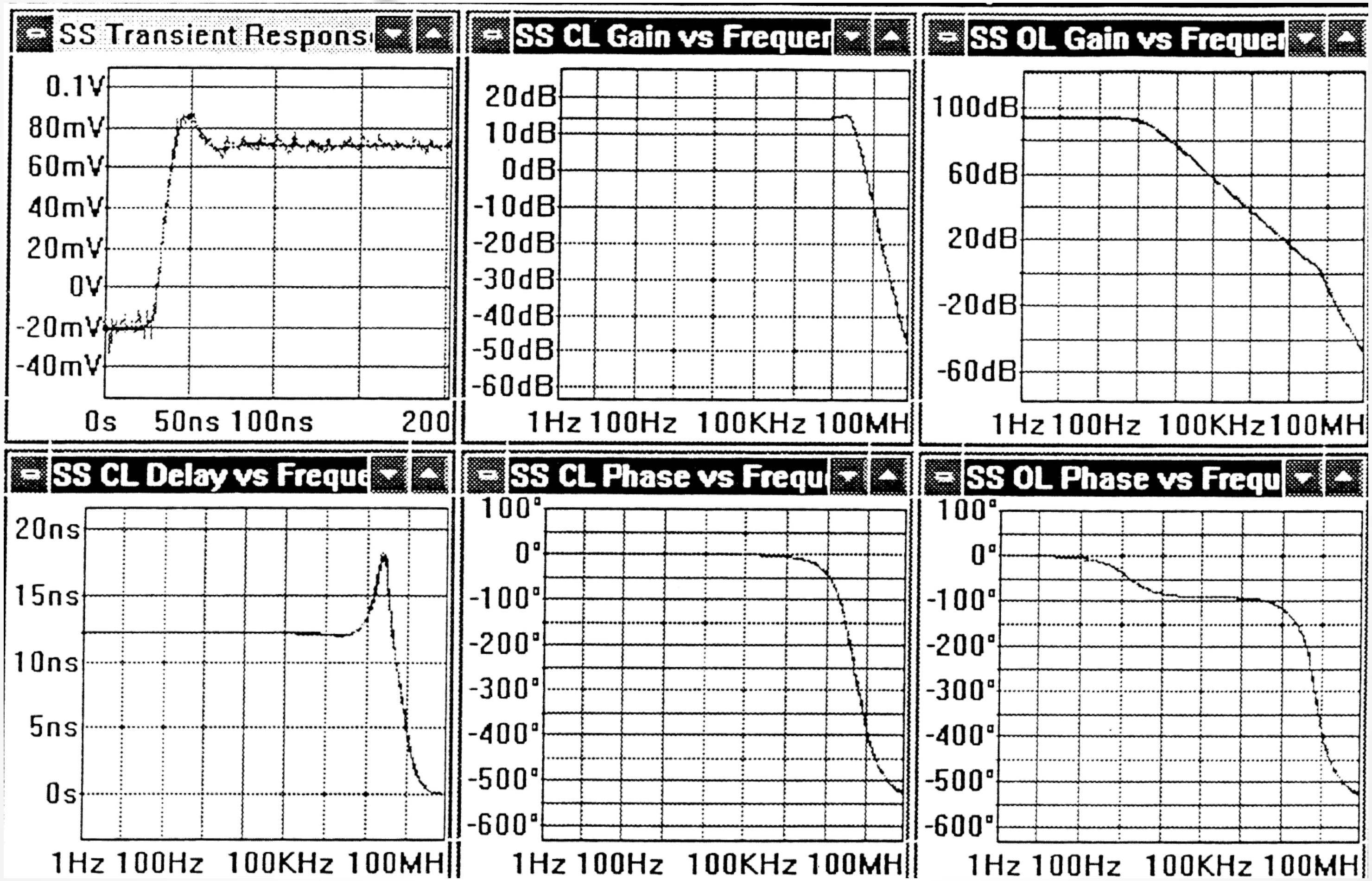
```
* OUTPUT STAGE
*
R49    49    99    18005
R50    49    50    18005
ISY    99    50    0.0058135
R61    60    99    73.2
R62    60    50    73.2
L61    60    52    1e-012
G63    63    50    113    60     0.0136612
G64    64    50    60     113    0.0136612
G65    60    99    99     113    0.0136612
G66    50    60    113    50     0.0136612
V61    61    60    3.38846
V62    60    62    3.38846
D61    113   61    DX
D62    62    113   DX
D63    99    63    DX
D64    99    64    DX
D65    50    63    DY
D66    50    64    DY
*
* MODELS AND END
*
.MODEL QX NPN(IS=1e-015 BF=2076)
.MODEL DX D(IS=1e-015)
.MODEL DY D(IS=1e-015 BV=50)
.ENDS
```

Build Strategy Demonstrated

- Sort (by magnitude) poles and zeros in four bins:
  - Real poles (lowest may be in early stage of operational amplifier)
  - Complex-conjugate poles
  - Real zeros
  - Complex-conjugate zeros
- Model complex-conjugate RHP zeros by all-pass network 14 or real RHP zeros by 13 and then in priority order use 9, 8, 7, and 6
- Model real zeros in priority order 5, 4, and 2
- Model remaining poles by 3 and 1
- Apply pole/zero cancellation for any “new” or “fixed” poles or zeros

Teraspeed Labs

Frequency Plots from Laplace Transform



“SS” is small signal

Conclusion

- SPICE macromodel generation strategy uses unity gain, cascaded RLC pole and zero stages
- Pole/zero cancellation is effective for adding RHP zeros
- Operational amplifier macromodel illustrates the process
- Process can be applied to any macromodel including those for high-speed applications