Speeding Up Transient Convolution with S-parameters

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Organization

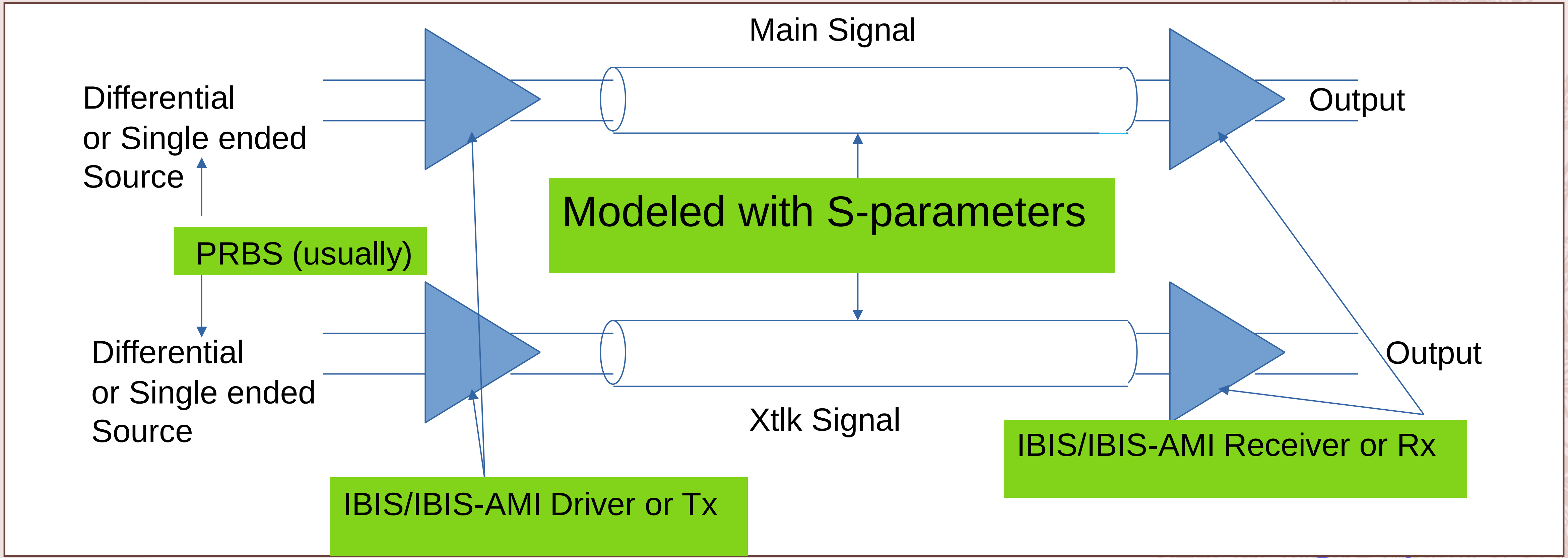
Why Convolution as opposed to Vector Fitting

Approximation Region of Convolution Integral

Prony's method and decaying exponentials for approximation

Examples

Schematic of Transient/Channel Simulation with IBIS/IBIS-AMI



Why Convolution is a better alternative to Vector Fitting

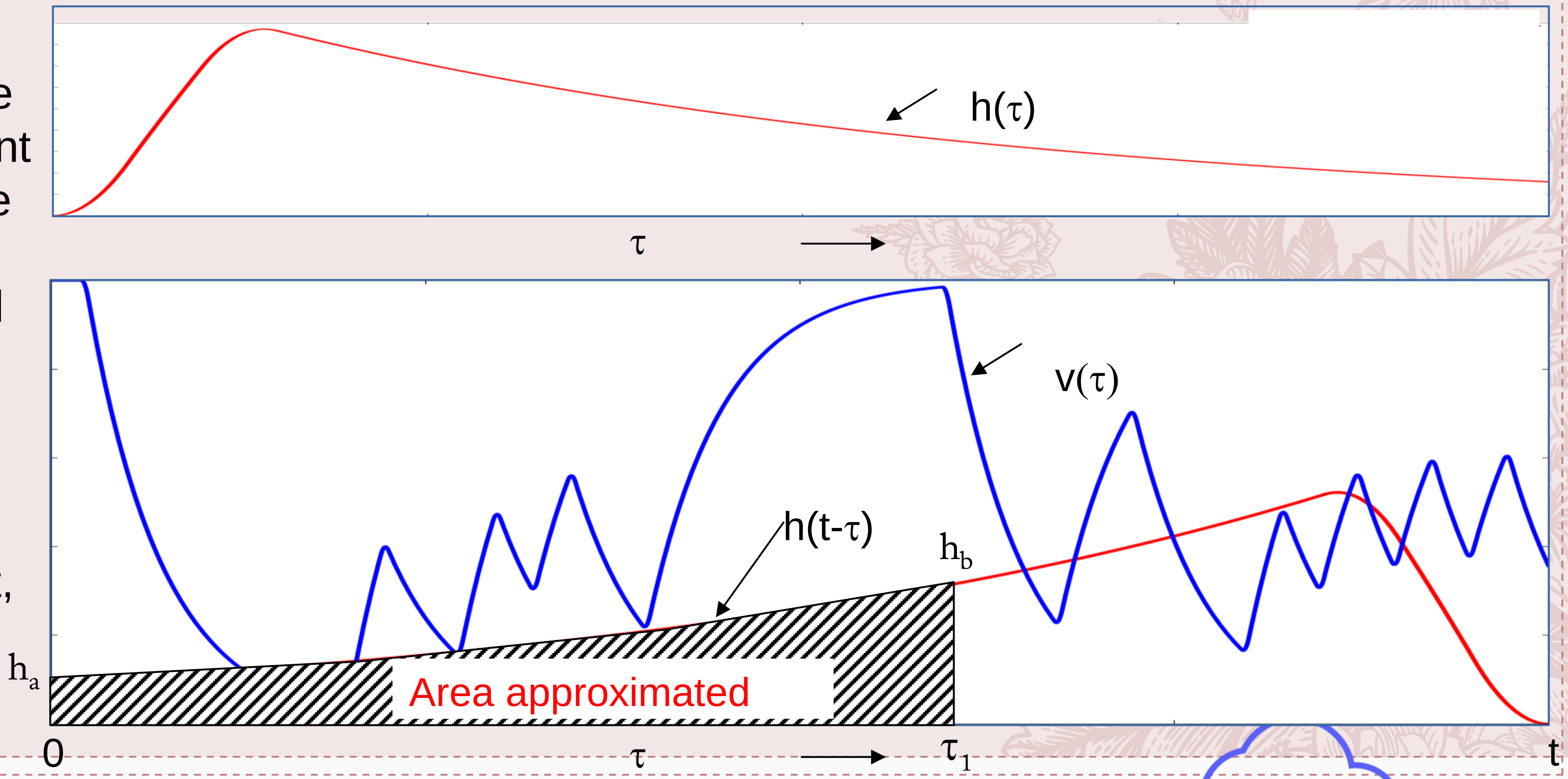
- Vector Fitting models S-parameters as rational functions. Passivity not guaranteed and enforcing passivity is time consuming. When simulation takes minutes (as for channel simulation or stateye), fitting alone may take an hour or hours, especially with large number of ports.
- For IBIS-AMI, Tx backend and Rx frontend linear parts are given as impulse responses. Easier to combine this with impulse response of channel.
- Delays are modeled more naturally with convolution approach and convolution is inherently more accurate
- Main disadvantage of convolution is that simulation is slow and varies as $O(n^2)$ with the number of timepoints n. So, simulating up to 1ns may be fast but simulating to 10ns will be 100 times slower. This paper aims to improve this.

Graphical representation of Convolution Integral

Let h(τ) be the impulse response of some S-parameter component s_{ij}. . Then the response to some input v(τ) at some time t is given by the convolution integral

$$I = \int_{-\infty}^{\infty} v(\tau)h(t-\tau)d\tau$$

In practice, integral is from 0 to t,



Approximation methodology

- We borrow concepts from Fast Multipole method. If the area of the impulse to be integrated is near the tail end, it can be approximated. The idea is that the tail end of the impulse has less effect at any given time t.
- Approximation is done in such a way that the dependence of h(t-τ) is separated as $h_1(t)*h2(\tau)$.
- An earlier paper (Kapur et. al.) used Lagrange polynomials for this. We found that decaying exponentials were a much better function to use for the following reasons:
  - *Passive structures are described by ODEs whose solutions are in the form of decaying exponentials. So, a natural choice.*
  - *Easy separation of t and τ* ➔ $\int_0^{t_1} v(\tau)h(t-\tau)d\tau = e^t \int_0^{t_1} v(\tau)\, e^{-\tau} d\tau$
  - *Works with lower orders and is not so sensitive to order choice.*
  - *Can be implemented conveniently using Prony's method.*
  - *Can be viewed as an alternative form of fitting. Fitting leads to poles and zeros which also are decaying exponentials in the time domain*

Equations with complex exponentials

Convolution integral with complex exponentials,

$$\int_0^{\tau_1} v(\tau)h(t-\tau)d\tau = \int_0^{\tau_1} v(\tau)\sum_{i=1}^{N}\frac{1}{2}A_i(e^{j\emptyset_i}e^{\lambda^+{}_i(t-\tau)} + e^{-j\emptyset_i}e^{\lambda^-{}_i(t-\tau)})d\tau$$

$$= \sum_{i=1}^{N}\frac{1}{2}A_i(e^{j\emptyset_i}e^{\lambda^+{}_i(t)}\int_0^{\tau_1} v(\tau)e^{\lambda^+{}_i(-\tau)}\,d\tau + \sum_{i=1}^{N}\frac{1}{2}A_i(e^{-j\emptyset_i}e^{\lambda^-{}_i(t)}\int_0^{\tau_1} v(\tau)e^{\lambda^-{}_i(-\tau)}\,d\tau$$

Again, the integral itself can be calculated once per section and reused. Note the amplitudes are complex conjugates of each other. So are the complex exponentials. The final results are always real.

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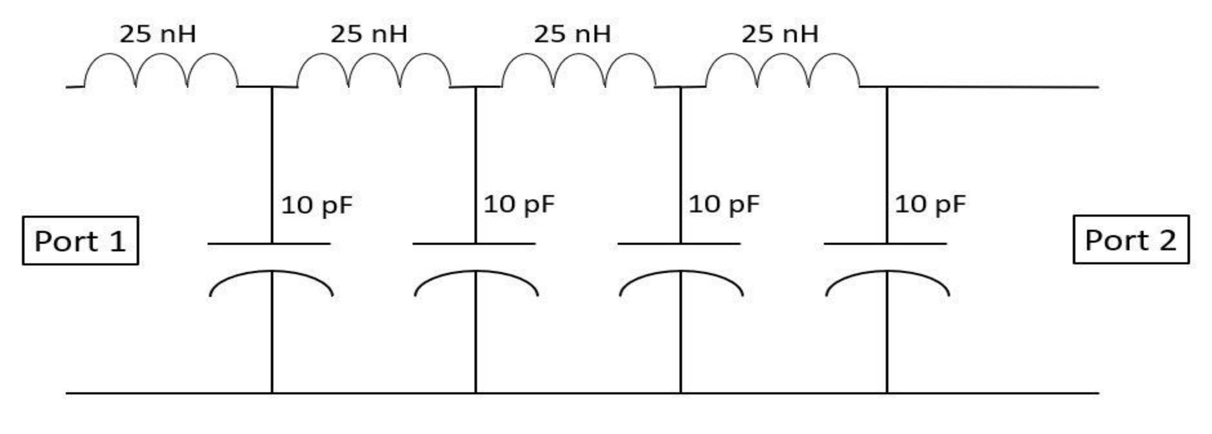
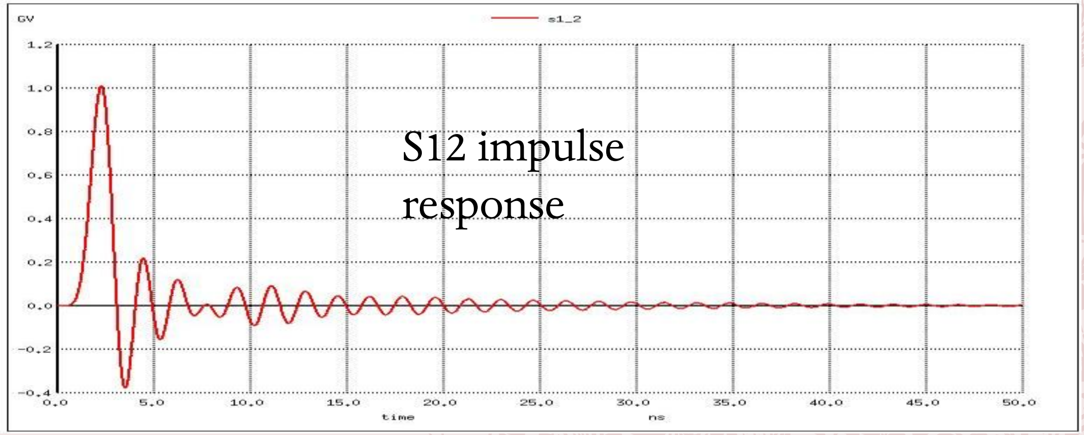
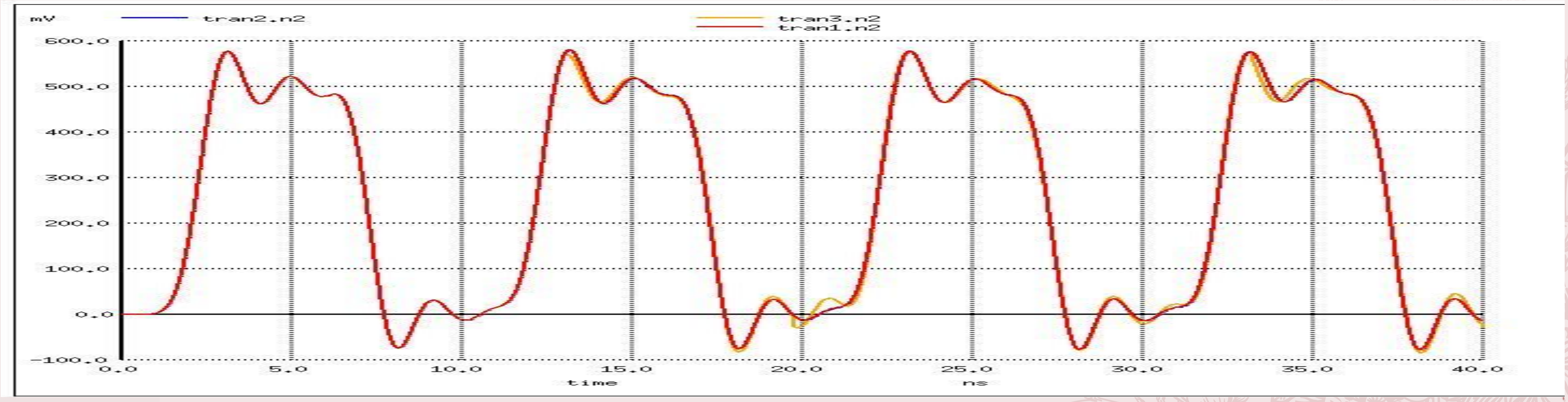
Example of lumped transmission line





This is a 50-ohm t-line of length 0.1 m. As it is a lumped model, the impulse response shows a lot of ringing.

Results for lumped transmission line with complex exponentials



With complex exponentials, even order of 2 (orange) gave a passable fit Results with order 8 were within simulation tolerance. Results compared to direct SPICE simulation

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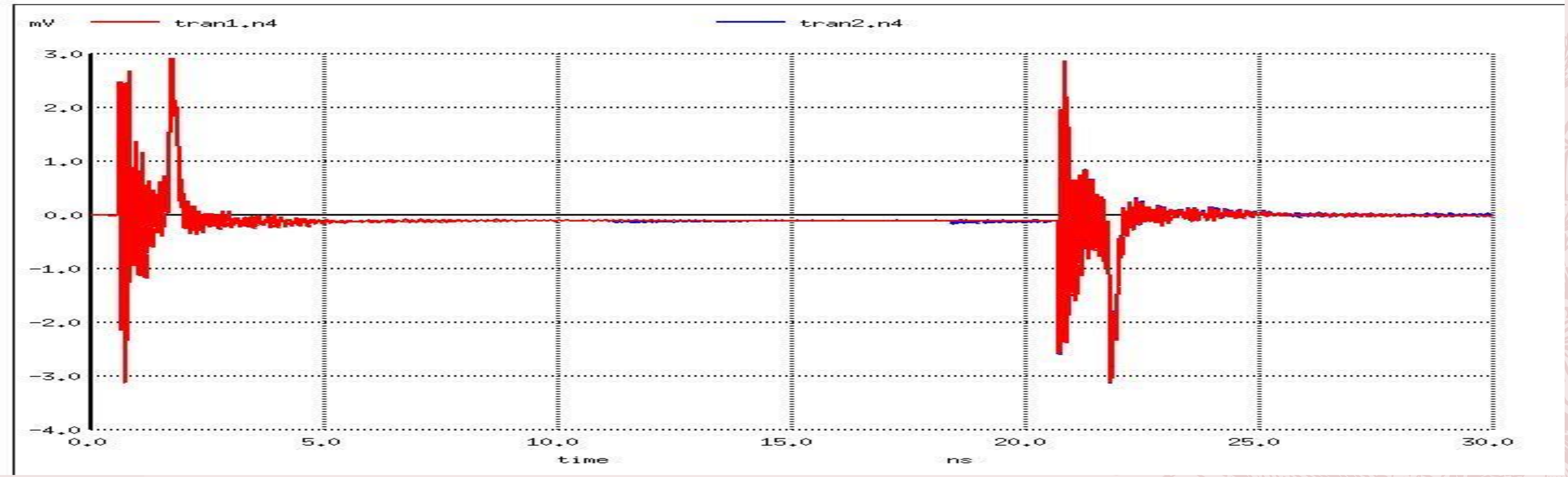
Results for more complicated example

This was a 4-port for a differential pair in a real-life PCB used for PCIexpress. The S-parameters were extracted using EM solvers. The PCB is multilayer and the traces go through more than one layer, vias, etc. Resulting S-parameters and impulse response have a lot of structure or detail.

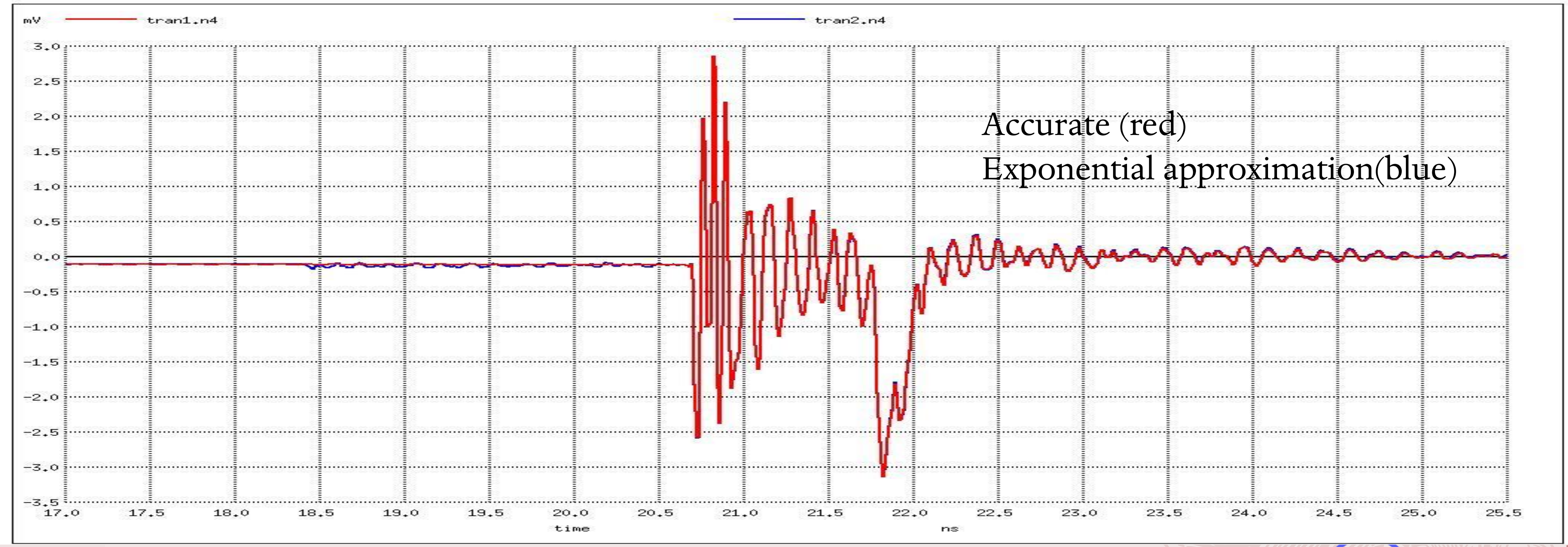
For purposes of this paper, the differential pair was simulated as single ended. One trace was excited and the other was the victim line. All ports were terminated in 50 ohms.

The near end and far end waveforms of the victim line are shown in the next two slides. Order 16 was used here with 5 sections. Speed-up was 5.5x with simulation up to 30 ns. For 40 ns, it was 14.8x

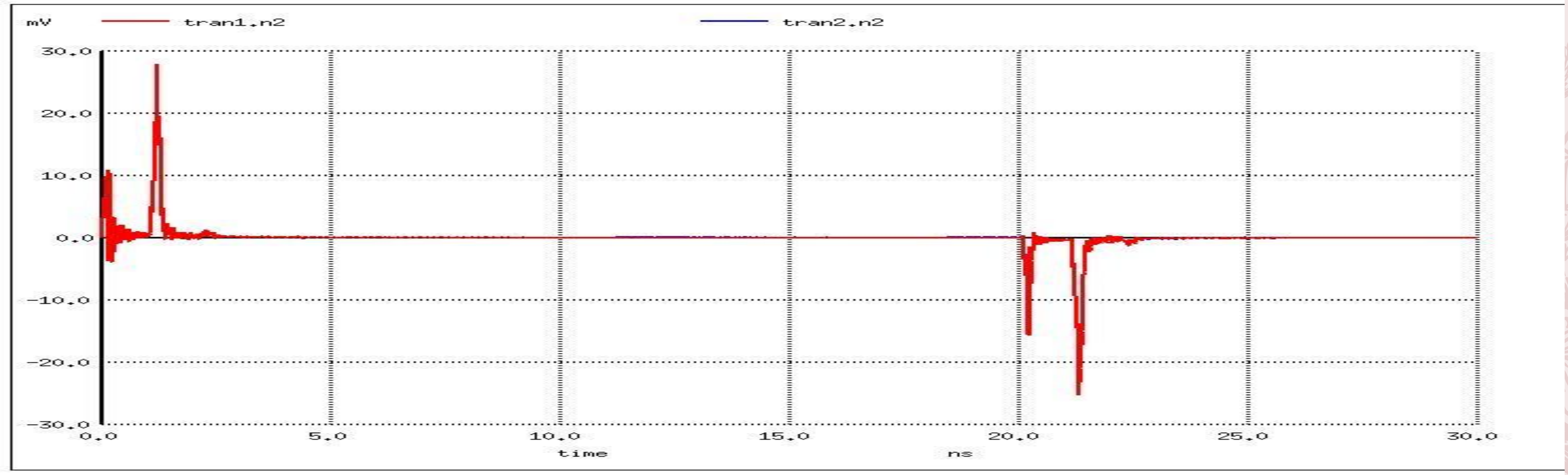
Far end cross-talk results for more complicated example



Far end cross-talk results zoomed in



Near end cross-talk results



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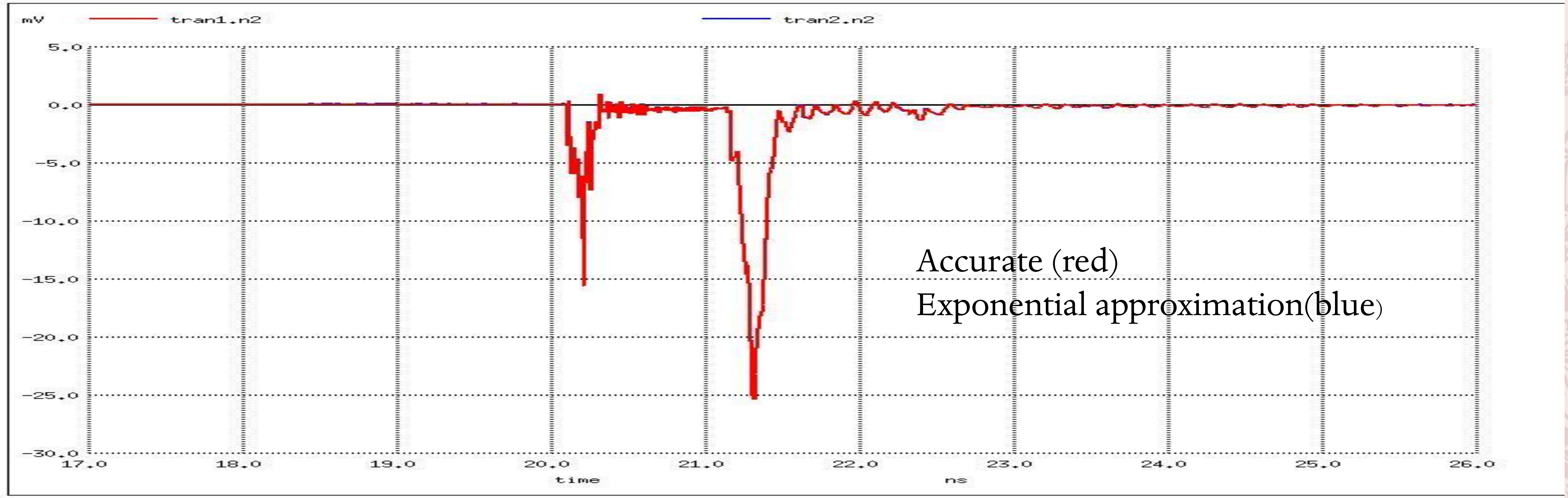
Near end cross-talk results zoomed in



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Conclusions

- It is shown that the convolution approach to transient simulation with S-parameters can be speeded up by over an order of magnitude using Fast Multipole type methods.

- Convolution can be a better approach to fitting both in terms of performance and accuracy. Fitting with a large number of ports has a huge initial overhead, especially with passivation. Convolution should be the preferred method for stateye or channel simulation.

- It is shown that complex exponentials are a good way to approximate the tail of impulse responses and can be optimized using Prony's method.

- Implementation has been done in NGSPICE so that the method is applicable with any general circuit.