Qualification of tabulated scattering parameters

Stefano Grivet Talocia

Politecnico di Torino, Italy
IdemWorks s.r.l.
stefano.grivet@polito.it

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Why S-parameters

- S-parameters are always defined
  - Impedance or admittance may not
- S-parameters are normalized
  - Good numerical properties in simulation
- S-parameters are easily measured
  - Even at very high frequency, good reliability
- Standard format for S-parameters
  - Touchstone files from measurement hardware
  - All field solvers provide S-parameters on output
- Tabulated frequency data
  - Intrinsic IP protection for vendors
  - Do not disclose design details, but only I/O electrical properties
- Best way to represent broadband EM/circuit interactions
  - The essence of Signal and Power Integrity
- Is this characterization complete?
  - Yes, but…
Scattering network functions

\[ \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} = \begin{bmatrix} S_{11}(s) & S_{12}(s) \\ S_{21}(s) & S_{22}(s) \end{bmatrix} \begin{bmatrix} A_1 \\ A_2 \end{bmatrix} \]

Scattering matrix

power waves

complex frequency
For circuits: real rational functions

\[ S_{ij}(s) = \frac{a_0 + a_1 s + \cdots + a_n s^n}{b_0 + b_1 s + \cdots + b_n s^n} \]

- For lumped circuits: S-parameters are real rational functions
- Valid for all complex frequencies in the entire complex plane

\[ S(s) = \begin{bmatrix} -sCR_0 & 2 \\ 2 + sCR_0 & -sCR_0 \\ 2 + sCR_0 & 2 + sCR_0 \end{bmatrix} \]

\( R_0 = \) reference resistance
Examples of S-parameter data

Via array
12 ports

Wiring harness
8 ports

High-speed channel
18 ports
Examples of S-parameter data
From frequency to time-domain: impulse response

Inverse Fourier transform

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(j\omega)e^{j\omega t} d\omega \]

Inverse Laplace transform

\[ h(t) = \frac{1}{2\pi j} \int_{\sigma_0-j\infty}^{\sigma_0+j\infty} S(s)e^{st} ds \]
Finding impulse responses

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} S(j\omega)e^{j\omega t} d\omega \]

\[ \omega_m = j \frac{2\pi m}{N\Delta} \]

\[ h(k\Delta) = \frac{1}{M} \sum_{m=0}^{M-1} S(j\omega_m) \exp\left( j \frac{2\pi mk}{M} \right) \]

Strategy 1:
Discrete Fourier Transform
(Fast Fourier Transform, FFT)

Strategy 2:
Fit a parametric model allowing analytic Fourier/Laplace inversion

\[ S(j\omega) \approx \sum_{n=1}^{N} \frac{R_n}{j\omega - p_n} + S_\infty \]

\[ h(t) \approx \sum_{n=1}^{N} R_n \exp(p_n t)u(t) + S_\infty \delta(t) \]
Qualification process

- The limited information in the Touchstone file must...
  - describe the electrical behavior of the structure of interest
  - have enough resolution: sampling
  - cover a sufficient bandwidth
  - fulfill fundamental passivity requirements
    - causality
    - energy gain, passivity

- Need a qualification methodology...
  - based on rigorous theoretical foundation
  - allowing robust numerical implementation
  - checking all above conditions
Passivity conditions

\[ S(-j\omega) = S^*(j\omega) \]

Guarantees real-valued impulse response. Always assumed by construction.

\[ \|S(j\omega)\| \leq 1 \quad \text{or} \quad \max_i \sigma_i \{S(j\omega)\} \leq 1 \]

Energy condition: structure must not amplify signals. Sometimes called simply “passivity” condition.

\[ S(j\omega) \text{ is causal} \]

No anticipatory behavior in time-domain. Note: causality is a prerequisite for passivity!
Passivity: a ping-pong match

The poor man’s illustration of passivity: iterate through signal reflections…

- Start with $B=0$ and $A_0=1$
- Model hits signal: $B_0 = S*A_0$
- Load hits signal: $A_1 = P*B_0 = (P*S)*A_0$
- Model hits signal: $B_1 = S*A_1 = S*P*S*A_0$
- Load hits signal: $A_2 = P*B_1 = (P*S)^2*A_0$
- ...
- And the winner is… $A_N = (P*S)^N*A_0$

$$A = \sum_{N=0}^{\infty} A_N$$
Passivity: a ping-pong match

Model: $B = S \cdot A$
Load: $A = P \cdot B$

One-port case

$$A_N = (PS)^N A_0$$

$|P| < 1, \quad |S| < 1 \quad \Rightarrow \quad A^N \text{ remains bounded}$

$|P| = 1, \quad |S| > 1 \quad \Rightarrow \quad A^N \xrightarrow{N \to \infty} \infty \quad \text{Blow-up!}$

$P$ is a reflection coefficient: for a passive load it does not exceed 1
Passivity: a ping-pong match

Model: \( B = S \cdot A \)

Load: \( A = P \cdot B \)

Passivity requires that \( |S(j\omega)| \leq 1 \) for all frequencies!

(not just the modeling bandwidth... all means really all, from 0 to Inf)
Passivity: what?

In case of matrices, math is more complicated…

… but visualization is simple and straightforward
Passivity: what?

\[ S(s) \text{ is passive} \Rightarrow \{\text{singular values of } S(j\omega)\} \leq 1, \forall \omega \]
Not all S-parameter models should be passive

Small-signal characterization of a FET-based amplifier
A passive interconnect model

All curves are below 1
Where do passivity violations come from?

- Data from measurement
  - Improper calibration and de-embedding
  - Human mistakes
  - Measurement noise

- Data from simulation
  - Poor meshing
  - Inaccurate solver
  - Bad models or assumptions on material properties
  - Poor data post-processing algorithms
  - Human mistakes
  - Putting together results from two solvers
Non-passive data: so what?

- Fitting Non-passive data → Non-passive model
  - Accurate
  - Model may blow-up during transient analysis

- Fitting Non-passive data → Passive model
  - Passivity enforcement
  - Acceptable if passivity violations are “small”
Can we tolerate a passivity violation?

**Measured data**
CM filter from vendor

**VERTICAL ZOOM**
Can we tolerate a passivity violation?

Measured data
Bad calibration by student

NO

S. Grivet-Talocia, “Qualification of tabulated scattering parameters”, IBIS Summit, 12 May 2010, Hildesheim, Germany
Although the responses present no energy gain, an artificial clipping is evident at high frequencies.

Can we fix passivity violations?

Test case: measured cable

Is this clipping ok?
Causality qualification

Much more tricky…
Causality: definitions

Time-domain

\[ H(j\omega) \]

Frequency-domain

Hilbert transform

Kramers-Krönig dispersion relations

\[
\begin{align*}
U(\omega) &= \frac{1}{\pi} \text{pv} \int_{-\infty}^{+\infty} V(\omega') \frac{d\omega'}{\omega - \omega'} \\
V(\omega) &= -\frac{1}{\pi} \text{pv} \int_{-\infty}^{+\infty} U(\omega') \frac{d\omega'}{\omega - \omega'}
\end{align*}
\]

\[ H(j\omega) = U(\omega) + jV(\omega) \]

Note: no delay extraction here
Causality: definitions

### Time-domain

- Input (IN)
- Output (OUT)
- Function: $H(j\omega)$

### Frequency-domain

- Hilbert transform
- Kramers-Krönig dispersion relations

$$H(j\omega) = \frac{1}{j\pi} \text{pv} \int_{-\infty}^{+\infty} H(j\omega') \frac{d\omega'}{\omega - \omega'}$$

- Self-consistency

  All samples are strongly related

Note: no delay extraction here
Causality check via dispersion relations

\[ \Delta(j\omega) = H(j\omega) - \frac{1}{j\pi} \text{pv} \int_{-\infty}^{+\infty} H(j\omega') \frac{d\omega'}{\omega - \omega'} \neq 0 \]

But in practice reconstruction is difficult because data are:

1. known only up to a maximum frequency \( \Omega \)
2. tabulated

TRUNCATION ERROR

DISCREETIZATION ERROR

Errors may hide causality violations... or point out bogus ones!

Identification of causality violations must account for these errors.
Causality check (ideal)

\[ H(j\omega) \xrightarrow{\text{Hilbert transform}} H_{\text{REC}}(j\omega) \]
Causality check (ideal)

\[ H(j\omega) \rightarrow \text{Hilbert transform} \rightarrow H_{\text{REC}}(j\omega) \]

Causal
\[ H_{\text{REC}}(j\omega) = H(j\omega) \]

Non-causal
\[ H_{\text{REC}}(j\omega) \neq H(j\omega) \]

Never verified numerically!
Causality check

\[ H(j\omega) \]

Generalized Hilbert transform

\[ H_{REC}(j\omega) \]

\[ \Delta(j\omega) \]

"Error bar"

Causal

\[ |H_{REC}(j\omega) - H(j\omega)| < \Delta(j\omega) \]

Non-causal

\[ |H_{REC}(j\omega) - H(j\omega)| > \Delta(j\omega) \]
Truncation error

\[
|E_n(j\omega)| = \frac{\prod_{q=1}^{n} (\omega - \bar{\omega}_q)}{j\pi} \text{pv} \int \frac{H(j\omega') - \beta(j\omega')}{\prod_{q=1}^{n} (\omega' - \bar{\omega}_q)} \frac{d\omega'}{\omega - \omega'} \quad \left|\omega'\right| > \Omega
\]

\[\leq T_n(\omega)\]

Analytic bound

Truncation error can be arbitrarily reduced

Full control of truncation effects!
Causality check results

Computed by IdEM R2009b
Causality check results

Computed by IdEM R2009b
Causality check error

Difference between reconstructed and raw data

\[ E_{dB}(j\omega) = 20 \log_{10} \left( \frac{|H_{REC}(j\omega) - H(j\omega)|}{\Delta(j\omega)} \right) \]

Numerical resolution ("error bar")
Causality check error

Maximum Causality Check Error (dB)

-20 -15 -10 -5 0 5 10 15 20 25 30

Frequency [GHz]

S(1,1)

Causal ( <0dB )

Non-causal ( >0dB )

Computed by IdEM R2009b
Where do causality violations come from?

- Data from measurement
  - Improper calibration and de-embedding
  - Human mistakes
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- Data from simulation
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Non-causal data: so what?

- **Non-causal data** → Fitting → **Bad accuracy**
  - Stable poles
  - **Model is not representative of the real structure**

- **Non-causal data** → Fitting → **Unstable poles**
  - Good accuracy
  - **Model cannot be simulated (will blow-up)**
Several causality violations are detected. Such violations are very large and spread throughout the entire band, especially for the diagonal responses of the S-matrix (return losses at all ports).
Due to the data inconsistencies, it is impossible to obtain an accurate model after the fitting process.

Note that the model accuracy is very poor, especially for the responses where the largest causality violations have been detected.
Causality violations in the data affect the fitting. The approximation error does not decrease even if a very large number of poles is used.

Output model is:
- stable
- passive
- causal
- not accurate
Removing the stability condition during the fitting (allowing poles in the right half plane), the fitting converges with good accuracy, but the final model is unstable.

Output model is:
- accurate
- not stable
- not causal
- not passive

Max Err: 0.006
Sampling: a trivial case

\[ S_{21}(s) = \frac{s^2 + \omega_0^2}{s^2 + \alpha s + \omega_0^2} \]

\[ \omega_0 = \frac{1}{\sqrt{LC}} \]

\[ \alpha = \frac{R_0}{2L} \]

- \( R_0 \) = reference resistance
- \( C = 5 \, \text{pF} \)
- \( L = 2 \, \text{nH} \)
Sampling: a trivial case

\[ C = \text{5 pF}, \quad L = \text{2 nH}, \quad R_0 = 50 \Omega \]

\[ \Omega \]

\[ 5 \times 10^2 \quad 5 \times 10^3 \quad 5 \times 10^4 \]

\[ S_{21}, \text{magnitude} \]

\[ \text{Frequency, GHz} \]

\[ 10^{-2} \quad 10^{-1} \quad 10^0 \quad 10^1 \quad 10^2 \]
Sampling: a trivial case

$\Omega = 10$

$R_L = \frac{1}{10^2}$

$C = 5 \text{ pF}$

$L = 2 \text{ nH}$
Sampling: a trivial case

\[ C = 5 \text{ pF} \]
\[ L = 2 \text{ nH} \]
\[ R_0 = 0.01 \Omega \]
Sampling: a trivial case

How do we expect a solver will interpret the blue samples?

![Graph showing S21 magnitude vs Frequency in GHz]
Detecting undersampled data via “causality” check

- **Undersampled**
  - Causality “violation” detected

- **Well sampled**
  - No causality “violations” detected
Conclusions

- Tabulated S-parameter data may hide serious issues
  - Passivity (energy gain) violations
    - Easily checked at individual frequencies (singular value test)
  - Causality violations
    - Can be detected using Generalized Hilbert Transform
    - Theoretically sound
    - Robust numerical implementation
  - Bad or insufficient sampling
    - Also detectable via Generalized Hilbert Transform

- If not detected (and corrected)
  - Any of these issues will lead to problems in simulation
    - With any tool or method (see SPI 2010 tutorial)
Thank you

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