Recent Developments on Advanced Macromodeling by Politecnico di Torino

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Signal and Power Integrity

Courtesy: Dr. Gianni Signorini, (formerly) Intel
The objective: behavioral models

**Behavioral models**: used extensively and successfully to approximate...

Many different modeling techniques (such as IBIS standards) are available

The most appropriate approach is application-dependent

We will focus here on passive structures and partially on analog devices
Behavioral models are intrinsically:

• Simplified and accurate descriptions
• Very general and design-independent
  • black-box: do not unveil details of the underlying design

Additional desirable features:

• **Compliance** with spice-like circuit solvers or other common simulation environments
• **Compliance with fundamental properties** of the reference system (e.g. stability and passivity)
• Generated automatically by non-expert users
Macromodels of passive structures

input

output

Courtesy Prof. Swaminathan, GA Tech
Macromodels of passive structures

Passive structures dynamics are properly described by linear ODEs or PDEs

Geometry, materials

Scattering data $\hat{S}_k = \hat{S}(j\omega_k)$

Extraction
EM simulation
Ckt simulation

LTI systems are best represented by rational transfer functions

Rational fitting
Passivity enforcement

Realization or synthesis

$\dot{x} = Ax + Bu$
$y = Cx + Du$

$S(s) = \sum_{n=1}^{N} \frac{R_n}{s - p_n} + S_\infty$
The approximations are topology-free and can potentially reproduce a variety of behaviors typical of distributed-parameter systems.

$$H(s) = \sqrt{\frac{s + 0.001}{s + 100}}$$

Non-rational smooth function

$H(j\omega)$
• Recent advancements on macromodeling by Politecnico di Torino
  • Macromodeling of large-scale systems (hundreds of I/O ports)
    • Speaker: Marco De Stefano, PhD candidate
    • Compression strategies
    • Fast passivity verification and enforcement
  • Parameterized (multivariate) macromodels
    • Speaker: Alessandro Zanco, PhD candidate
    • Model structure and enhanced scalability (Radial Basis Functions – RBF)
    • Stability enforcement
  • Small-signal modeling of (nonlinear) analog circuit blocks
    • Speaker: Tommaso Bradde, PhD candidate
    • Embedding bias-dependence through parameterized macromodels
    • Theoretical assessment of dissipativity
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    • **Theoretical assessment of dissipativity**
Handling large-scale LTI systems

Large-Scale LTI Systems

PCBs

Shielding enclosures

PDNs

Dynamic Order

Input/output Ports

TLs
Handling large-scale LTI systems

For structures with many electrical ports, the behavior can be recovered as a linear combination of a reduced number of case-dependent basis functions obtained from a data-compression technique (e.g. truncated SVD)

Surrogate Macromodeling → Data Compression Technique + Vector Fitting

- Accurate and Fast
- Data compression based on Singular Value Decomposition (SVD)
- Robust and reliable: full control over accuracy

Accuracy OK, Stability OK, Passivity ???
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Handling large-scale LTI systems: passivity

- Passivity check based on a Hamiltonian matrix eigenvalues computation is the state-of-the-art
- Such technique becomes infeasible when the model complexity grows, in terms of
  - P: Ports
  - N: Number of Poles
- Computational cost scales as $O((kPN)^3)$

A new procedure to address large-size macromodels is required
→ adaptive sampling in the frequency domain may be sufficient...

64 GB of RAM necessary to check passivity of 400-port, 90-pole model
Adaptive-sampling-based passivity checking scheme developed to overcome the complexity of Hamiltonian checks

Based on a 2-stage approach

1. Pole-based adaptive frequency warping
2. A passivity-driven tree-search divide-and-conquer strategy

Fast variations of eigenvalues (or singular values) trajectories are mostly induced by the model poles resonances

The result is a fast and reliable strategy for the passivity characterization of large-scale macromodels.

Very small passivity violations can be effectively detected!

Frequency Warping = Change of variable
\[ \zeta = W(f) \]

Handling large-scale LTI systems: passivity

Massive testing campaign to assess the algorithm reliability

- All examples run on 8GB of RAM laptop (64GB are required for the Hamiltonian Check)
- Large size cases speed-up from 10 to 100X
- Small and medium size performances are comparable

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Parameterizing macromodels

- **Parametric sweep**
- **Simulation or Measurement**

Multiple S-parameters

\[ \bar{H}_{k;m} = \bar{H}(j\omega_k; \vartheta_m) \]
\[ k = 1, \ldots, K; \ m = 1, \ldots, M \]

**Model construction**

\[ \dot{w} = A(\vartheta)w + B(\vartheta)u \]
\[ y = C(\vartheta)w + D(\vartheta)u \]

Parameterized model
Possible parameterization approaches

Approach #1: interpolate independent non-parametric “root” macromodels

\[ H(s, \vartheta_m) = \sum_{n=1}^{N} \frac{R_n^m}{s - p_n^m} + H_\infty^m \]

“Root” models in pole-residue form (each with individual poles and residues)

Obtained by interpolating the closest “root” models

Pros:
- Very simple scheme
- Interpolation schemes exist that guarantee passivity

Cons:
- Model is fully defined only by including a large set of “root” macromodels
- Poles are usually many more than required
Possible parameterization approaches

Approach #2: embed in closed the parameter variability in the model

\[ H(s, \vartheta) = \frac{N(s; \vartheta)}{D(s; \vartheta)} \]

Rational parametric model

Pros:
• Compact parametric model  (few kB)
• No need to generate a possible large number of “root” models
• SPICE - compatible

Cons:
• Advanced techniques are required to guarantee stability and passivity
Rational model structure

\[ H(s; \theta) = \frac{N(s; \theta)}{D(s; \theta)} = \sum_n \sum_\ell \frac{N_{n,\ell}(\theta) \varphi_n(s)}{\sum_n \sum_\ell R_{n,\ell}(\theta) \varphi_n(s)} \]

To be estimated

\[ \frac{N(s; \theta)}{D(s; \theta)} \approx \tilde{H}(j\omega_k; \theta_m) \]

Partial Fractions \( \varphi_n(s) : \frac{1}{s - q_n} \)

Parameter basis function \( \xi_\ell(\theta) : \)

High-dimensional kernel (RBF) expansion

Gaussian kernel \( \xi_\ell(\theta) = e^{-\varepsilon \| \theta - \theta_\ell \|^2} \)

PRO:
Good scalability in high-dimensions

Handling many parameters

Model complexity

Exponential

Linear!

# of parameters

Standard

RBF

12 May 2021  2021 European IBIS Summit @ SPI 23/41
The model $H(s; \vartheta)$ depends upon some free hyper-parameters, whose number:

<table>
<thead>
<tr>
<th>STANDARD</th>
<th>RBF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scales exponentially</td>
<td>Fixed!!!</td>
</tr>
</tbody>
</table>

Gaussian kernel

$$\xi_\ell(\vartheta) = e^{-\varepsilon \| \vartheta - \vartheta_\ell \|^2}$$

- Shape parameter
- # of RBFs
- RBF’s centers

There exists techniques to optimize these hyper-parameters for model compactness and accuracy


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Multivariate models: fast stability enforcement

\[ H(s; \theta) = \frac{N(s; \theta)}{D(s; \theta)} = \sum_n \sum_l R_{n,l} \xi_l(\theta) \phi_n(s) = \sum_n \frac{R_n(\theta)}{s - p_n(\theta)} \]

For uniform stability

\[ \Re\{D(s; \theta)\} > 0, \ \forall \ s, \ \theta \]

\[ \Re\{p_n(\theta)\} < 0, \ \text{for all} \ \theta \]

Implicit poles parameterization

NO CONTROL ON POLES LOCATIONS!

Parameter space sampling

... but, with positive-definite kernels ...

$$\Re\{D(s; \boldsymbol{\vartheta})\} = \Re\{\sum_n \sum_l r_{n,l} \xi_l(\boldsymbol{\vartheta}) \varphi_n(s)\} > 0$$

For real poles $q_n$

$$r_{n,l} > 0$$

And for complex poles $\alpha_n \pm j\beta_n$

$$-\alpha_n r'_{n,l} \pm \beta_n r''_{n,l} > 0$$

Analytic stability constraints!
Multivariate models: fast stability enforcement

A parameterized microstrip transmission line

10-independent parameters
- Inner lines length
- Stubs length
- Load resistances
- Substrate electrical parameters

It required 14 minutes to extract a STABLE model, with accuracy $11 \times 10^{-3}$
A 10-parameters example

Low Noise Amplifier

Automatic generation of a STABLE parameterized model: 6 minutes!

Model vs Data error: $9.2 \cdot 10^{-3}$
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Affine linearized models of NL analog blocks

**Nonlinear equations**

\[ \dot{w} = F(w, u) \]
\[ y = G(w, u) \]

**Small signal analysis**

Linearization around the operating point

**Local model**

\[ \dot{w} = Aw + Bu \]
\[ y = Cw + Du \]

**PROBLEM:** post-layout circuit equations are unavailable (hidden)

**SOLUTION:** data driven modeling approach based on AC analysis

**Rational Approximation** (Vector Fitting)

\[ \dot{x} = Ax + Bu \]
\[ y = Cx + Du \]
Affine linearized models of NL analog blocks

\[
y = \tilde{y} + Y_0
\]

\[
u = \tilde{u} + U_0
\]

\[
\dot{x} = Ax + Bu
\]

\[
y = Cx + Du + Y_C
\]
Operating point parameterization

$P$ electrical ports

$P$ DC inputs set the operating point

$U_0 = \begin{bmatrix} U^1_0 \\ \vdots \\ U^P_0 \end{bmatrix}$

Standard rational fitting techniques return models for each single OP

$U^2_0$  

The bias space of interest

$U^1_0$  

model for the entire bias space

$U^2_0$  

model for all the $U_0$ values of interest

---

For active circuit blocks: $\vartheta \equiv U_0$

Parametric sweep

Simulation or Measurement

Frequency response data

$\tilde{H}_{k,m} = \tilde{H}(j\omega_k; \vartheta_m)$

$k = 1, \ldots, K; \ m = 1, \ldots, M$

Multivariate Rational Fitting

Stability

SPICE netlist

State-space realization

$\dot{x} = A(\vartheta)x + B(\vartheta)u$

$y = C(\vartheta)x + D(\vartheta)u + Y_C(\vartheta)$

Circuit synthesis

Parameterized model

$H(s; \vartheta) = \frac{N(s; \vartheta)}{D(s; \vartheta)}$
Example: a post-layout voltage regulator

\[ V_{DD} \in [0.9, 1.1] \text{V} \quad I_L \in [0, 10] \text{mA} \]

\[ U_0 = \begin{bmatrix} V_{DD} \\ I_L \end{bmatrix} \quad \text{Induce the parameterization} \]

\[ \mathbb{H}_m = \mathbb{H}(j\omega; \theta_m) \]
Transient simulation results: multitone signal

Bias point: $V_{DD} = 1V$, $I_L = 5mA$

Small-signal: multitone noise of amplitude $120mV$ overimposed to $V_{DD}$

Computed time span: $100ms$

Regulated Voltage

Model time requirements: 363 ms

Transistor level post Layout: 258 s

SPEED UP FACTOR: 700X
Transient simulation results: sequential pulses

Bias point: $V_{DD} = 1V$, $I_L = 5mA$

Small-signal: Sequential square pulses of amplitude $\pm 25mV$ over $V_{DD}$

Computed time span: $100\mu s$

Model time requirements: 93 ms

Transistor level post Layout: 63 s

Regulated Voltage

SPEED UP FACTOR: 675X

Dashed Red Lines : Model
Black lines: post layout LDO
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Remark on dissipativity of active circuits

Any integrated circuit is not able to generate energy on its own

Proper circuit functioning is allowed by external power supply

Illustrative Example: 3-P Amplifier

The amplifier fullfills the dissipation inequality:

\[
\frac{\partial E(t)}{\partial t} \leq p_1(t) + p_2(t) - p_3(t)
\]

\(E(t)\): Stored energy

\(p_i(t)\): i-th power flow

Energetic contraints reflects into NL dynamics (e.g. saturation)
For linear affine systems we have:

\[ p(t) = (U_0 + \tilde{u})^T (Y_0 + \tilde{y}) \quad \text{AND} \quad E(x) = \frac{1}{2} x^T P x + q^T x + c, \quad P = P^T \]

NEW RESULT: DISSIPATIVITY OF LINEAR AFFINE SYSTEMS

\[ \exists P, q: \dot{\tilde{z}}^T \Sigma(P) \tilde{z} + 2\theta_0(P, q, U_0, Y_0)^T \tilde{z} - 2U_0^T Y_0 \leq 0 \]

Additional terms due to different input power

\[ p(t) = U_0^T Y_0 + U_0^T \tilde{y} + \tilde{u}^T Y_0 + \tilde{u}^T \tilde{y} \]

DC power: \[ U_0^T Y_0 \] \quad Positive

Small signal power: \[ \tilde{u}^T \tilde{y} \] \quad Undefined sign

Cross-power: \[ U_0^T \tilde{y} + \tilde{u}^T Y_0 \] \quad Undefined sign

Conclusions

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