



# K.T. Wang (Wang Algebra) – Expanded History

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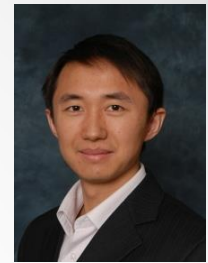


# Agenda

- **Credit**
- **Wang Algebra**
- **T-coils**
- **Wang's Biography**
- **References**



# Credit: Dr. Cong Ling



- **Dr. Cong Ling (Imperial College, London, UK) provided the historical information and Chinese links for this presentation**  
<https://www.imperial.ac.uk/people/c.ling>
- **While researching quaternions (below), Ling discovered Ross' reference to K.T. Wang in the Asian IBIS Summit presentation [7], Beijing, 2007 (slides 3 and 4 on next two pages)**
- **The comment by Duffin seemed incomplete and did not convey K.T. Wang's full story**
- **From uploaded links in Chinese sites, a very brief expanded history shows that K.T. Wang was a brilliant researcher, administrator, and philosopher**
- **Quaternion rules:  $f(a,b,c,d) = a+bi+cj+dk$ ;  $i^2=j^2=k^2=-1$ ;  $ijk=-1$** 
  - **Used for astronautics, robotics, computer visualization, animation, special effects in movies, navigation, etc.**



# Wang Algebra – 70+ Years Ago

**K.T. Wang, “On a new method of analysis of electrical networks,” in *Memoirs 2, Nat. Res. Inst. Eng. Academia Sinica*, pp. 1-11, 1934**

S.L. Ting, “On the general properties of electrical network determinants,” *Chinese J. Physics*, vol 1, pp. 18-40, 1935

C.T. Tsai, “Short cut methods of Wang algebra of network problems,” *Chinese J. Physics*, vol. 3, pp. 141-181, 1939

R.J. Duffin and T.D. Morley, “Wang algebra and matriods,” *IEEE Trans Circuit and Systems*, vol CAS-25, no 9, pp. 755-762, Sept., 1978

W.K. Chen, *Graph Theory and Its Engineering Applications* (ch. 5, sect. 4, “The Wang-algebra formulation”), World Scientific Publ., 1997

**Wang Algebra:**

$$\mathbf{XX} = \mathbf{0}$$

$$\mathbf{X} + \mathbf{X} = \mathbf{0}$$

$$\mathbf{XY} = \mathbf{YX}$$

=

$$*\mathbf{W}*$$



# Wang Algebra

**“K. T. Wang managed an electrical power plant in China, and in his spare time sought simple rules for solving the network equations. Wang's rules were published in the reference indicated below [5]. Wang could not write in English so his paper was actually written by his son, then a college student. Raoul Bott and I recognized that Wang's rules actually define an algebra. We restated the rules as three postulates for an algebra:**

$$xy = yx, x + x = 0, xx = 0.”$$

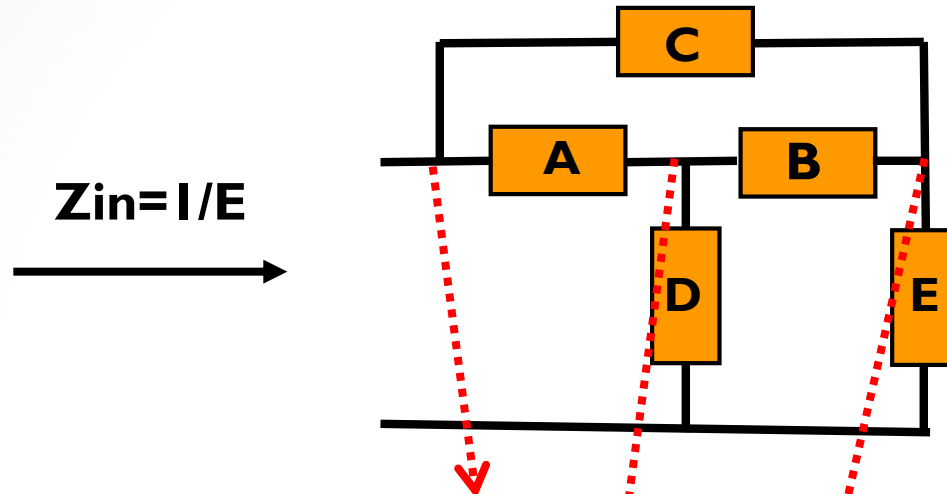
R.J. Duffin, “Some Problems of Mathematics and Science,” Bulletin of the American Mathematical Society, Nov. 1974, p. 1060

(Use Google to search for this reference.)

(“[5]” is the K.T.Wang reference on previous slide)



# Solving $[I]=[Y][V]$ for $Z_{in}$ (Traditional Method)



**Nodal Equations:**

**A ... E are  
admittances**

$$\mathbf{E} = \mathbf{I}/\mathbf{R}$$

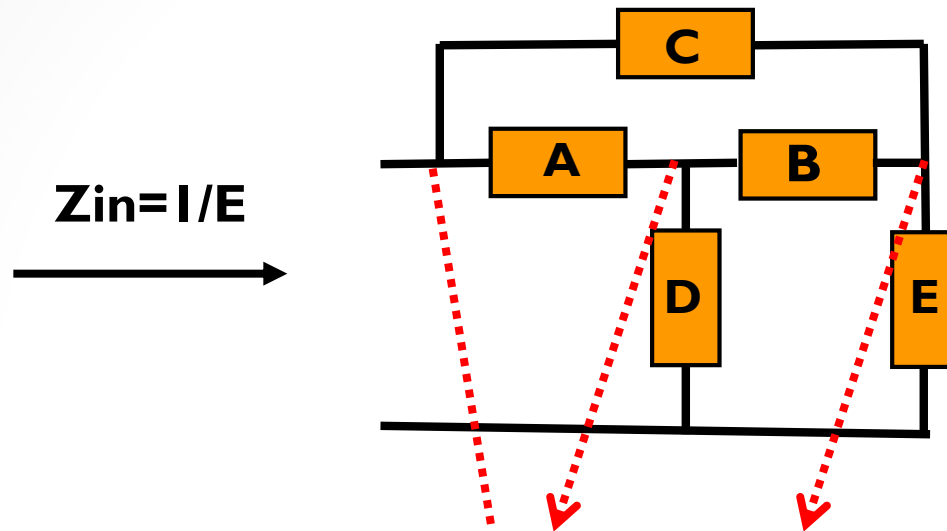
$$[\mathbf{I}] = [\mathbf{Y}][\mathbf{V}] = \begin{bmatrix} I_{in} \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} A+C & -A & -C \\ -A & A+B+D & -B \\ -C & -B & B+C+E \end{bmatrix} \begin{bmatrix} V_{in} \\ V_D \\ V_E \end{bmatrix}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{\begin{vmatrix} A+B+D & -B \\ -B & B+C+E \end{vmatrix}}{\begin{vmatrix} A+C & -A & -C \\ -A & A+B+D & -B \\ -C & -B & B+C+E \end{vmatrix}} = \frac{AB + AC + AE + BC + BD + BE + CD + DE}{ABD + ABE + ACD + ACE + ADE + BCD + BCE + CDE}$$

**(18 initial terms yields 8  
final denominator terms)**



# Solving $[I]=[Y][V]$ for $Z_{in} = R$ (Wang Algebra for Nodal Equations)



**Nodal Equations:**

**A ... E are  
admittances**

**$E = I/R$**

$$Z_{in} = \frac{\text{numerator}}{\text{denominator}} = \frac{(A + B + D) * W * (B + C + E)}{(A + C) * W * (\text{numerator})} = \frac{1}{E}$$

**XX=0**



$$AB + AC + AE + \cancel{BB} + BC + BE + BD + CD + DE$$

**X+X=0**



$$\cancel{ABC} + ABE + ABD + ACD + ADE + \cancel{ABC} + ACE + BCE + BCD + CDE$$

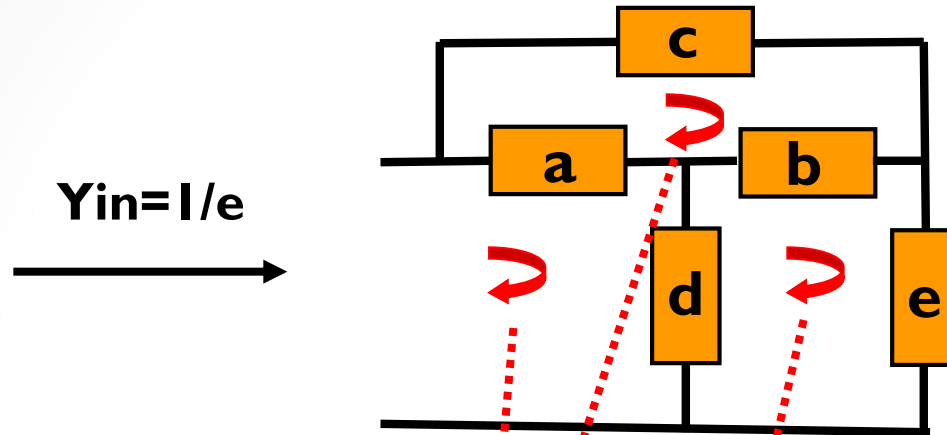
(after

**XX=0)**

$$\frac{AB + AC + AE + BC + BD + BE + CD + DE}{ABD + ABE + ACD + ACE + ADE + BCD + BCE + CDE}$$



# Solving $[V]=[Z][I]$ for $Z_{in}= I/Y_{in} = R$ (Wang Algebra for Loop Equations)



Loop Equations:

a ... e are impedances

e = R

$$Y_{in} = \frac{\text{numerator}}{\text{denominator}} = \frac{(a + b + c) * \boxed{W} * (b + d + e)}{(a + d) * \boxed{W} * (\text{numerator})} = \boxed{\frac{1}{e}}$$

~~XX=0~~ →

$$= \frac{ab + ad + ae + \cancel{bb} + bd + be + bc + cd + ce}{\dots}$$

~~X+X=0~~ →

$$= \frac{\cancel{abd} + abe + abc + acd + ace + \cancel{abd} + ade + bde + bcd + cde}{\dots}$$

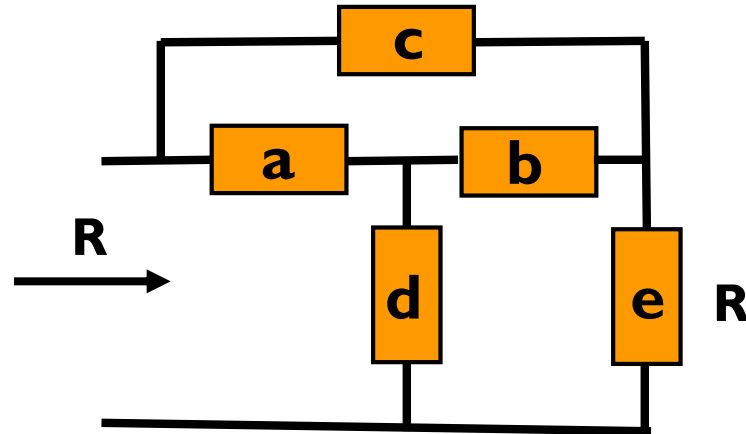
(after  
~~XX=0~~)

$$= \frac{ab + ad + ae + bd + be + bc + cd + ce}{abc + abe + acd + ace + ade + bde + bcd + cde}$$





# Constant R Constraint



## General

$$d(a + b) + ab + R(a - b) - R^2 - \frac{R^2(a + b)}{c} = 0$$

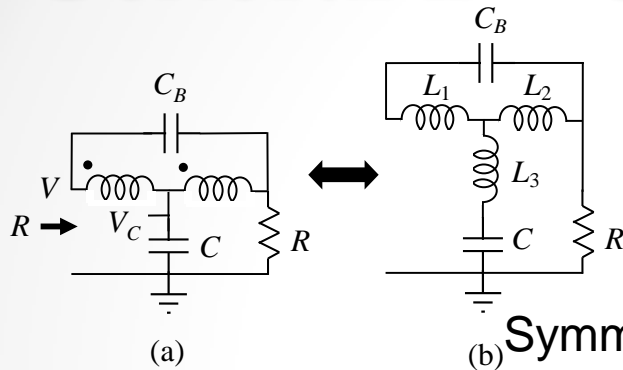
## Symmetric (a = b)

$$2da + a^2 - R^2 - \frac{2R^2a}{c} = 0$$

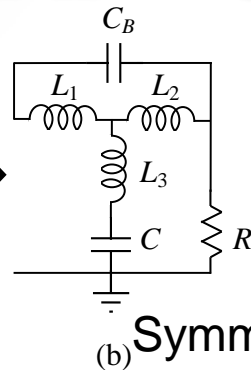
**Substitute impedances and equate powers of the Laplace variable “s” for constant R relationships**



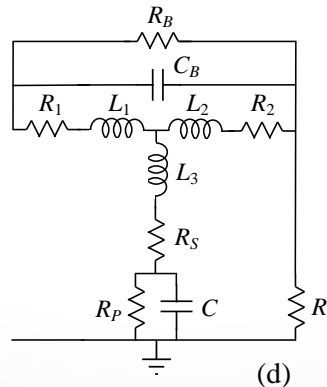
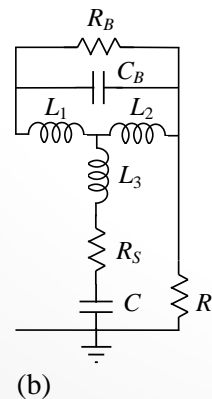
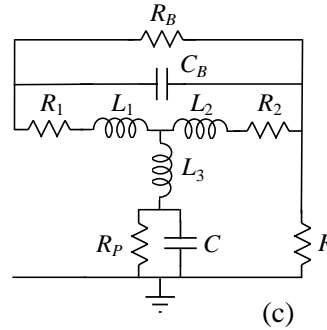
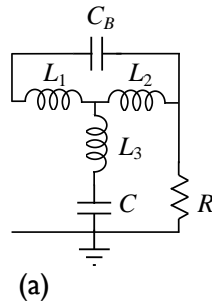
# Wang Algebra Used for Deriving General 2<sup>nd</sup> Order T-coil Equations



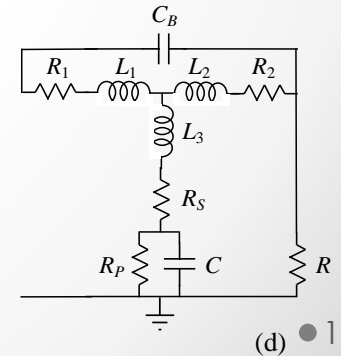
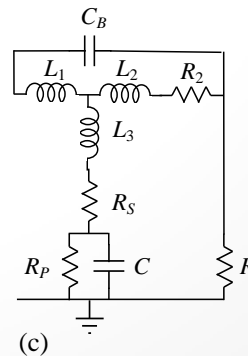
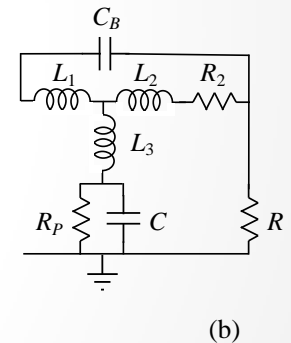
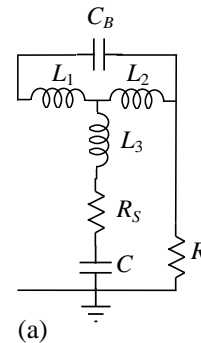
Standard T-coil and equivalent model for coupled inductors



## Symmetric T-coils

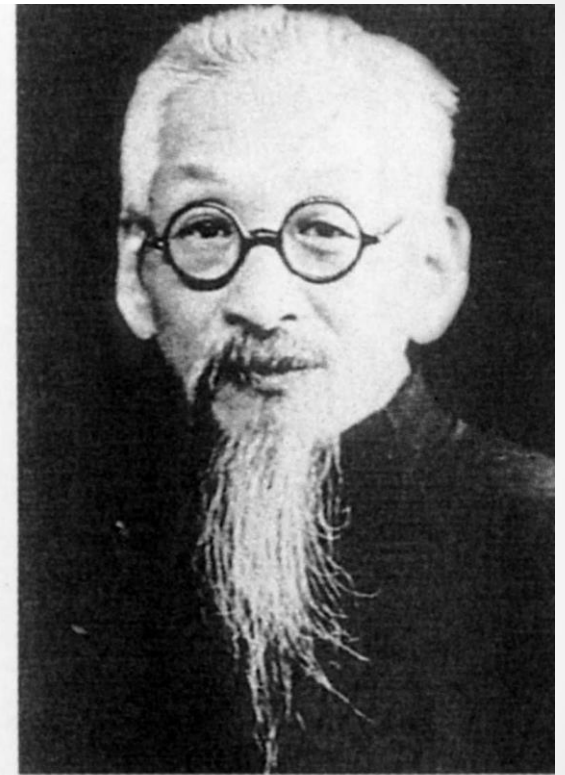


## Asymmetric T-coils



# Ki-Tung Wang (王季同)

**Ki-Tung Wang (王季同, 1875—1948) was a Chinese mathematician, electrical engineer and philosopher. Believed to be the first Chinese mathematician to publish a paper in an international journal, he is well known for his work on Wang Algebra, as well as investigation on the relationship between sciences and Buddhism [1].**



王守竞之父王季同



# Six Children

- **Shu-Zhen Wang (1899-1991), medical doctor from Johns Hopkins University, head of Shanghai Women's Hospital**
- **Shou-Jin Wang (1904-1984), Ph.D. in quantum physics from Columbia University, worked at Peking University and retired from MIT Lincoln Lab. <https://www.guokr.com/article/441034/>**
- **Ming-Chen Wang (1906-2010), Ph.D. in physics from University of Michigan, professor at Tsinghua University.**
- **Shou-Rong Wang (1917-1966), graduated from Tsinghua University, expert on semiconductors, professor at Nankai University/Tianjin University**
- **Shou-Wu Wang (1919-2014), Ph.D. from Purdue University, expert on semiconductors, fellow of Chinese Academy of Sciences**
- **Shou-Jue Wang (1925-2016), graduated from Tongji University, expert on semiconductors, fellow of Chinese Academy of Sciences**



# Ki-Tung Wang's Brief Biography (1)

- His ancestor, Ao Wang (王鏊, 1450-1524) ranked no. 3 in the Imperial Examination; later became a Grand Secretary of the Cabinet (equivalent to Prime Minister) of the Ming Dynasty [2].
- His father Song-Wei Wang (王颂蔚, 1849-1895) also a Jinshi (Imperial Scholar), the highest degree of Imperial Examination in ancient China.
- 1875: Born into a prominent family in Suzhou, Jiangsu Province
- 1895: Graduated from Tongwen Guan (同文馆), or Multilingual College (modern-day Peking University); hired as a mathematical lecturer there; had already published several Chinese articles on classical Chinese mathematics and modern mathematics
- 1909: Served as an administrator of Chinese students in Europe, then he did internships at the British Electrical Company and Siemens [1]. During this period, he published a paper on the differentiation of quaternionic functions in the Proceedings of the Royal Irish Academy [3], which is believed to be the first paper published by Chinese mathematicians in international journals [1]



# Ki-Tung Wang's Brief Biography (2)

- **1914: Went on to industry and became an engineer at the Zhenjiang Power Plant, Jiangsu Province [1]**
- **1928: Was appointed Research Fellow at the National Research Institute of Engineering, Academia Sinica (i.e., Chinese Academy of Sciences) [1]; proposed a new method to derive the impedance of electrical networks, which is sometimes advantageous to the traditional Kirchhoff law [4]**
- **Very interested in philosophy beyond the limits of modern sciences [8], [9], [10]; had several publications on sciences and Buddhism, including a book *Comparative study of Buddhism and Sciences* printed in 1933 and reprinted in 2014 [10]**
- **After retirement, devoted himself to Buddhism**



# Conclusions

- **Brief history shows Ki-Tung Wang was an accomplished mathematician, engineer, administrator, and philosopher**
- **Ki-Tung Wang may have known more English than stated, although a son with academic credentials may have helped write the Wang Algebra paper**
- **Wang Algebra is still relevant for general T-coil derivations**



# References (1)

- [1] 郭金海[Guo Jinhai] (2015), 王季同: 最早在国际刊物发表数学论文的中国学者 [K. T. Wang: First Chinese scholar to publish a mathematical paper in International journals]. Institute for the History of Natural Sciences, Chinese Academy of Sciences, [http://www.ihns.cas.cn/kxcb\\_new/kpwz\\_new/201602/t20160229\\_4538251.html](http://www.ihns.cas.cn/kxcb_new/kpwz_new/201602/t20160229_4538251.html)
- [2] [https://en.wikipedia.org/wiki/Wang\\_Ao\\_\(Grand\\_Secretary\)](https://en.wikipedia.org/wiki/Wang_Ao_(Grand_Secretary))
- [3] K. T. Wang, The Differentiation of Quaternion Function, Proceedings of the Royal Irish Academy. Vol. 29 (1911/1912), pp. 73-80.
- [4] K. T. Wang, On a new method of analysis of electrical networks, in Memoirs 2, Nat. Res. Inst. Eng. Academia Sinica, pp. 1-11, 1934.
- [5] R. J. Duffin, An analysis of the Wang algebra of networks, Trans. Amer. Math. Soc. 93 (1959), 114-131.
- [6] R. J. Duffin, Some Problems of Mathematics and Science, Bulletin of the American Mathematical Society, Nov. 1974, p. 1060.





# References (2)

- [7] B. Ross, Wang Algebra and Interconnects, Asian IBIS Summit Beijing, China, September 11, 2007.
- [8] K. T. Wang, Comparative study of Buddhism and Sciences, Shanghai Buddhism Press, 1933, Reprinted 2014.  
<http://www.nnycjd.com/jsrw/wjt/8000.html>.
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<http://www.nnycjd.com/jsrw/wjt/6374.html>.
- [10] K. T. Wang, Advise scholars all over the world to study Buddhism, 1943,  
<http://www.nnycjd.com/jsrw/wjt/7957.html>.
- [11] K. T. Wang, A method of finding the most economical ratio of transformer sizes.
- [12] K. T. Wang, A new formula for helical springs and a new graphic method for finding the area of irregular shapes.
- [13] K. T. Wang, Letter to Yan Li on classical Chinese mathematics vs. modern mathematics.

