K.T. Wang (Wang Algebra) – Updated Expanded History

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Agenda

- **Wang Algebra**
- **T-coils**
- **Wang’s Biography**
- **References**

Wang Algebra – Nearly 90 Years Ago

K.T. Wang, "On a new method of analysis of electrical networks," in Memoirs 2, *Nat. Res. Inst. Eng. Academia Sinica*, pp. 1-11, 1934

S.L. Ting, "On the general properties of electrical network determinants," *Chinese J. Physics*, vol 1, pp. 18-40, 1935

C.T. Tsai, "Short cut methods of Wang algebra of network problems," *Chinese J. Physics*, vol. 3, pp. 141-181, 1939

W.-L. Chow, "On electric networks," J. Chinese Math. Soc., vol. 2, pp. 321–339, 1940

R.J. Duffin and T.D. Morley, "Wang algebra and matroids," IEEE Trans Circuit and Systems, vol CAS-25, no 9, pp. 755-762, Sept, 1978

W.K. Chen, Graph Theory and Its Engineering Applications (ch. 5, sect. 4, "The Wang-algebra formulation"), World Scientific Publ., 1997

Wang Algebra:

$$\boxed{\begin{array}{l} XX = 0 \\ X+X = 0 \\ XY = YX \end{array}} \quad = \quad *\boxed{W}*$$

Wang Algebra

Theorem 1 (Wang Algebra). *Let* $\mathbf{A} = [a_{ij}]_{n\times n}$ *be a symmetric matrix, i.e.,* $a_{ij} = a_{ji}$, *where* $1 \le i, j \le n$. *Write the diagonal elements of* $\mathbf{A}$ *as*

$$a_{ii} = a'_{ii} - \sum_{j\neq i} a_{ij}.$$

Then the determinant $\det(\mathbf{A})$ *can be computed as*

$$\det(\mathbf{A}) = \prod_{i=1}^{n} \left(a'_{ii} - \sum_{j\neq i} a_{ij} \right) \tag{2}$$

in Wang algebra $\mathbb{W}$.

- Wang algebra gives a clever method to compute the determinant of a symmetric matrix.
- Remark: (2) needs to be computed symbolically.

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Proof

- A proof outline was already given in the original article of K.T. Wang.
- Duffin's proof was based on Grassmann algebra, which is 10-pages long.
- Chow's proof based on matrix theory is also quite tricky.
- In [Ross-Ling], we present a one-page proof using group theory.

Ross and Ling, "Wang Algebra: From Theory to Practice", *IEEE Open Journal of Circuits and Systems*, 2022
https://arxiv.org/pdf/2208.09649.pdf

Corollary 1 (Wang's Rule). *The determinant of a planar network does not contain any terms containing a square or a factor* 2. *Moreover, all its terms have coefficient* $+1$.

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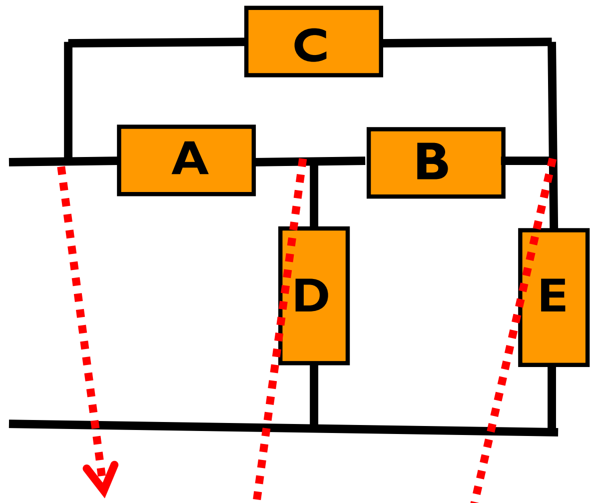
Solving [I] = [Y][V] for Zin (Traditional Method)

Zin=1/E



Nodal Equations:

A … E are admittances

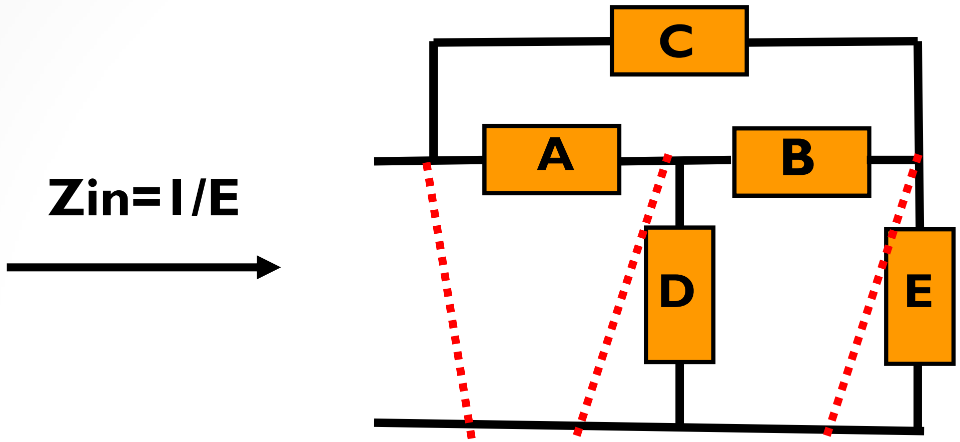
E = 1/R

$$[\mathrm{I}]=[\mathrm{Y}][\mathrm{V}]=\begin{bmatrix}\mathrm{Iin}\\0\\0\end{bmatrix}\begin{bmatrix}A+C & -A & -C\\-A & A+B+D & -B\\-C & -B & B+C+E\end{bmatrix}\begin{bmatrix}\mathrm{Vin}\\\mathrm{V_D}\\\mathrm{V_E}\end{bmatrix},$$

$$\mathrm{Zin}=\frac{\mathrm{Vin}}{\mathrm{Iin}}=\frac{\begin{vmatrix}A+B+D & -B\\-B & B+C+E\end{vmatrix}}{\begin{vmatrix}A+C & -A & -C\\-A & A+B+D & -B\\-C & -B & B+C+E\end{vmatrix}}=\boxed{\frac{AB+AC+AE+BC+BD+BE+CD+DE}{ABD+ABE+ACD+ACE+ADE+BCD+BCE+CDE}}$$

(18 initial terms yields 8 final denominator terms)

Solving [I] = [Y][V] for Zin = R (Wang Algebra for Nodal Equations)



Nodal Equations:

A … E are admittances

E = 1/R

$$\mathrm{Zin} = \frac{\text{numerator}}{\text{denominator}} = \frac{(A+B+D)*\boxed{\mathrm{W}}*(B+C+E)}{(A+C)*\boxed{\mathrm{W}}*(\text{numerator})} = \boxed{\frac{1}{E}}$$

XX=0 →

X+X=0 (after XX=0) →

$$= \frac{AB + AC + AE + \cancel{BB} + BC + BE + BD + CD + DE}{A\cancel{B}C + ABE + ABD + ACD + ADE + A\cancel{B}C + ACE + BCE + BCD + CDE}$$

$$= \boxed{\frac{AB + AC + AE + BC + BD + BE + CD + DE}{ABD + ABE + ACD + ACE + ADE + BCD + BCE + CDE}}$$

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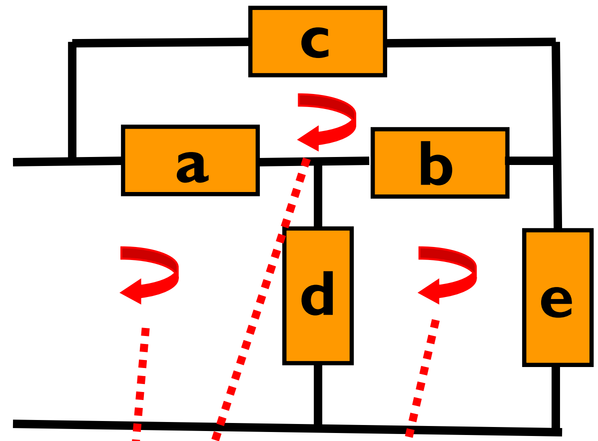
Solving [V] = [Z][I] for Zin = 1/Yin = R (Wang Algebra for Loop Equations)

Yin=1/e



Loop Equations:

a … e are impedances

e = R

$$\mathrm{Yin} = \frac{\text{numerator}}{\text{denominator}} = \frac{(a+b+c)*\boxed{\mathrm{W}}*(b+d+e)}{(a+d)*\boxed{\mathrm{W}}*(\text{numerator})} = \boxed{\frac{1}{e}}$$

XX=0 →

X+X=0 → **(after XX=0)**

$$= \frac{ab+ad+ae+\cancel{bb}+bd+be+bc+cd+ce}{\cancel{abd}+abe+abc+acd+ace+\cancel{abd}+ade+bde+bcd+cde}$$

$$= \boxed{\frac{ab+ad+ae+bd+be+bc+cd+ce}{abc+abe+acd+ace+ade+bde+bcd+cde}}$$

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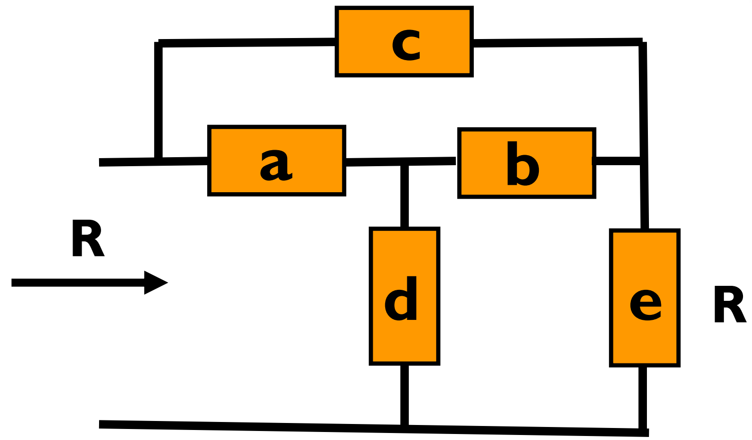
Constant R Constraint



General

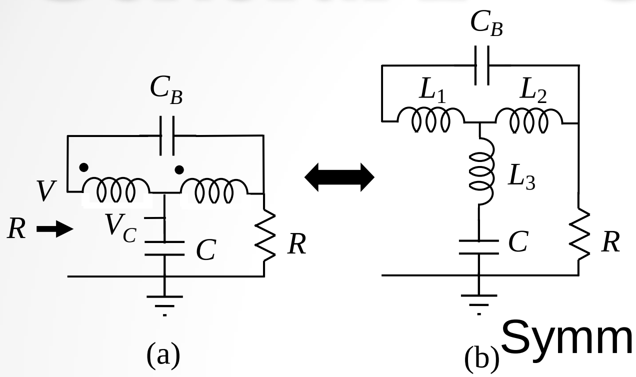
$$d(a+b)+ab+\mathrm{R}(a-b)-\mathrm{R}^2-\frac{\mathrm{R}^2(a+b)}{c}=0$$

Symmetric (a = b)

$$2da+a^2-\mathrm{R}^2-\frac{2\mathrm{R}^2a}{c}=0$$

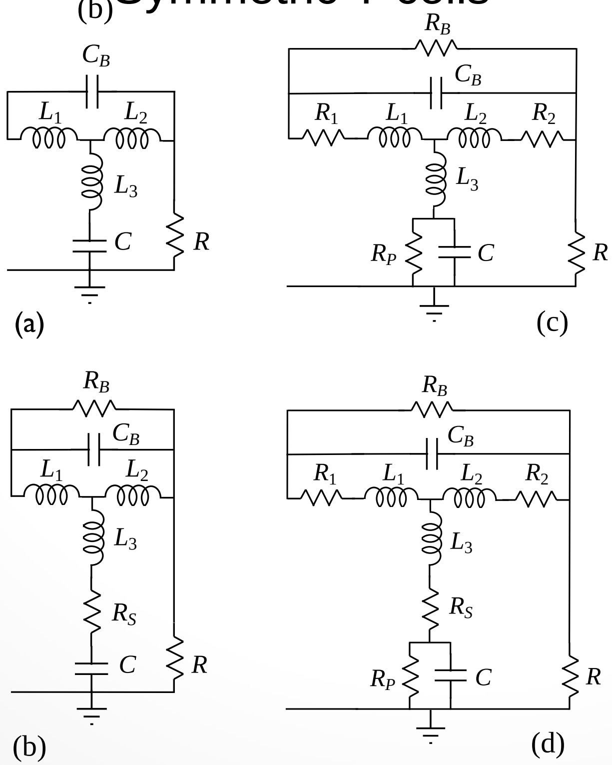
Substitute impedances and equate powers of the Laplace variable "s" for constant R relationships

Wang Algebra Used for Deriving General 2nd Order T-coil Equations

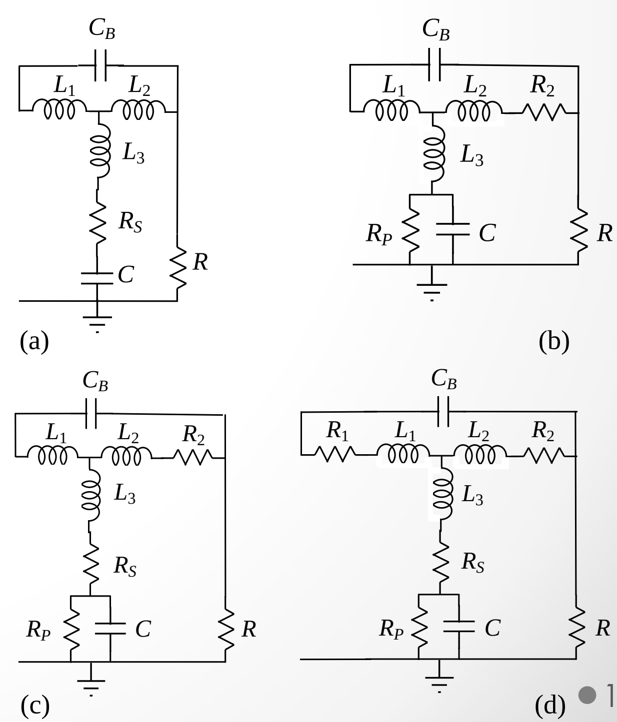


Standard T-coil and equivalent model for coupled inductors

Symmetric T-coils



Asymmetric T-coils



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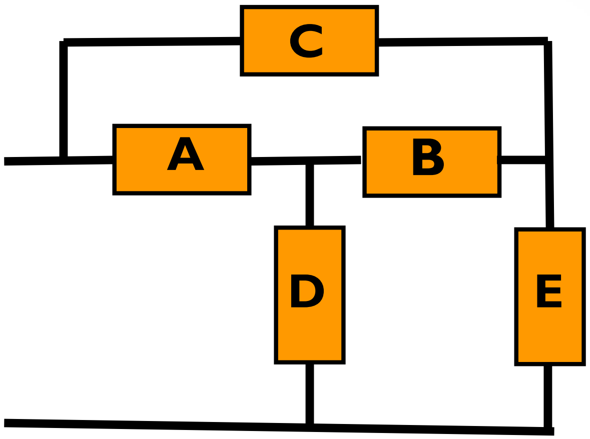
Application to Graph Theory



- Wang algebra also gives an algebraic method to enumerate the trees and cotrees of a graph.
- A (spanning) tree of a graph is a set of edges which connect all nodes and which do not contain any loops. For example, edges {A, B, E} form a tree.
- The complement of a tree in a graph in called a cotree. For example, {C, D} form a cotree.

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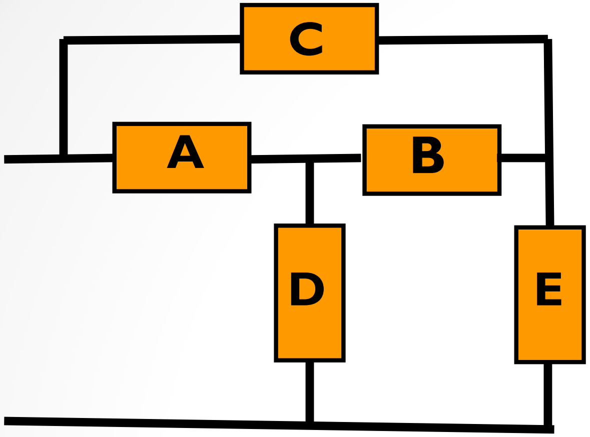
Enumerating Trees/Cotrees



$$\mathbf{S} = \begin{pmatrix} A+C & -A & -C \\ -A & A+B+D & -B \\ -C & -B & B+C+E \end{pmatrix}$$

$= \mathbf{S}_1$

- Cotrees

$$\det(\mathbf{S}_1) = (A+B+D)(B+C+E)$$
$$= AB + AC + AE + BC + BE + BD + CD + DE.$$

- Trees

$$\det(\mathbf{S}) = (A+C)\det(\mathbf{S}_1)$$
$$= ABE + ABD + ACD + ADE$$
$$+ ACE + BCE + BCD + CDE.$$

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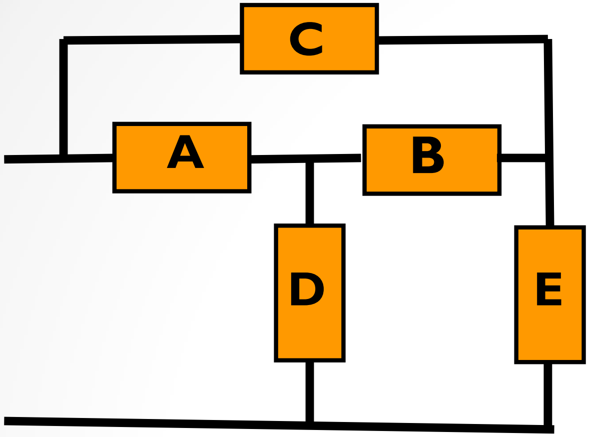
Counting Trees/Cotrees



$$\mathbf{S} = \begin{pmatrix} A+C & -A & -C \\ -A & A+B+D & -B \\ -C & -B & B+C+E \end{pmatrix}$$

$$\begin{pmatrix} A+B+D & -B \\ -B & B+C+E \end{pmatrix} = \mathbf{S}_1$$

- Setting A = B = C = D = E = 1, we obtain the number of trees/cotrees:
- det($\mathbf{S}$) = 8
- det($\mathbf{S}_1$) = 8

Wang Algebra

"K. T. Wang managed an electrical power plant in China, and in his spare time sought simple rules for solving the network equations. Wang's rules were published in the reference indicated below [5]. Wang could not write in English so his paper was actually written by his son, then a college student. Raoul Bott and I recognized that Wang's rules actually define an algebra. We restated the rules as three postulates for an algebra:

$$xy = yx,\ x + x = 0,\ xx = 0."$$

R.J. Duffin, "Some Problems of Mathematics and Science," Bulletin of the American Mathematical Society, Nov. 1974, p. 1060
(Use Google to search for this reference.)

("[5]" is the first reference on slide 3 and [4] on slide 19.)

Ki-Tung Wang (王季同)

Ki-Tung Wang (王季同, 1875－1948) was a Chinese mathematician, electrical engineer and philosopher. Believed to be the first Chinese mathematician to publish a paper in an international journal, he is well known for his work on Wang Algebra, as well as investigation on the relationship between sciences and Buddhism [1].

王守竞之父王季同

Ki-Tung Wang's Brief Biography (1)

- His ancestor, Ao Wang (王鏊, 1450-1524) ranked no. 3 in the Imperial Examination; later became a Grand Secretary of the Cabinet (equivalent to Prime Minister) of the Ming Dynasty [2].
- His father Song-Wei Wang (王颂蔚, 1849-1895) also a Jinshi (Imperial Scholar), the highest degree of Imperial Examination in ancient China.
- 1875: Born into a prominent family in Suzhou, Jiangsu Province
- 1895: Graduated from Tongwen Guan (同文馆), or Multilingual College (modern-day Peking University); hired as a mathematical lecturer there; had already published several Chinese articles on classical Chinese mathematics and modern mathematics
- 1909: Served as an administrator of Chinese students in Europe, then he did internships at the British Electrical Company and Siemens [1]. During this period, he published a paper on the differentiation of quaternionic functions in the Proceedings of the Royal Irish Academy [3], which is believed to be the first paper published by Chinese mathematicians in international journals [1]

Ki-Tung Wang's Brief Biography (2)

- 1914: Went on to industry and became an engineer at the Zhenjiang Power Plant, Jiangsu Province [1]
- 1928: Was appointed Research Fellow at the National Research Institute of Engineering, Academia Sinica (i.e., Chinese Academy of Sciences) [1]; proposed a new method to derive the impedance of electrical networks, which is sometimes advantageous to the traditional Kirchhoff law [4]
- Very interested in philosophy beyond the limits of modern sciences [8], [9], [10]; had several publications on sciences and Buddhism, including a book *Comparative study of Buddhism and Sciences* printed in 1933 and reprinted in 2014 [10]
- After retirement, devoted himself to Buddhism

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Conclusions

- **The comment by Duffin seemed incomplete and did not convey K.T. Wang's full story**
- **Brief history shows Ki-Tung Wang was an accomplished mathematician, engineer, administrator, and philosopher**
- **Ki-Tung Wang may have known more English than stated, although a son with academic credentials may have helped write the Wang Algebra paper**
- **Wang Algebra is still relevant for general T-coil derivations**

Ross and Ling, "Wang Algebra: From Theory to Practice", *IEEE Open Journal of Circuits and Systems*, 2022, https://arxiv.org/pdf/2208.09649.pdf

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[2] https://en.wikipedia.org/wiki/Wang_Ao_(Grand_Secretary)

[3] K. T. Wang, The Differentiation of Quaternion Function, Proceedings of the Royal Irish Academy. Vol. 29 (1911/1912), pp. 73-80.

[4] K. T. Wang, On a new method of analysis of electrical networks, in Memoirs 2, Nat. Res. Inst. Eng. Academia Sinica, pp. 1-11, 1934.

[5] R. J. Duffin, An analysis of the Wang algebra of networks, Trans. Amer. Math. Soc. 93 (1959), 114-131.

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