Zhejiang University

# Efficient PDN Optimization Method Based on Equivalent Circuit Fine-Tuning

Presenter： Li Jiang

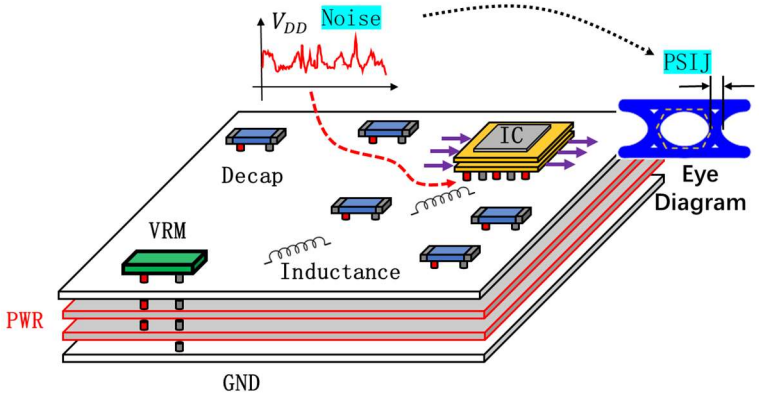
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Hangzhou, China
November 6, 2026

Zhejiang University College of Information Science and Electronic Engineering

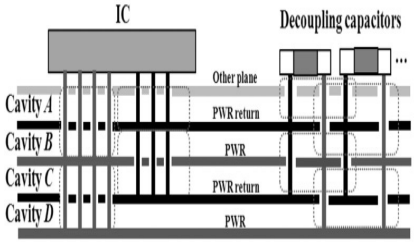
# Background：Power Integrity

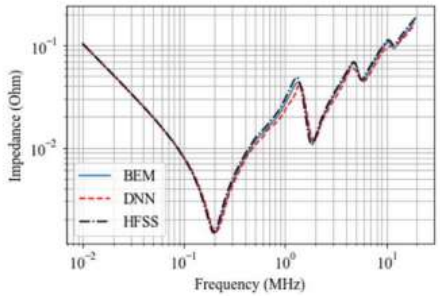
**Power Integrity（PI）：ensuring that power distribution network（PDN） adequately support the performance of an entire electronic system**

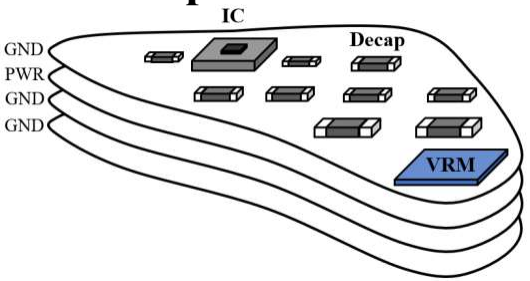


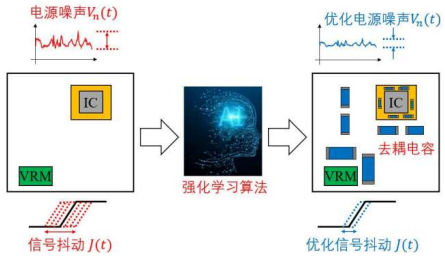

- Target：Reduce impedance and noise
- Works：1.Impedance modeling of PDN
  2.Decoupling capacitor layout
  3.PDN structure design and tuning

**PDN modeling** → **Impedance calculation** → **Decap Optimization** → **Time-Domain characters**



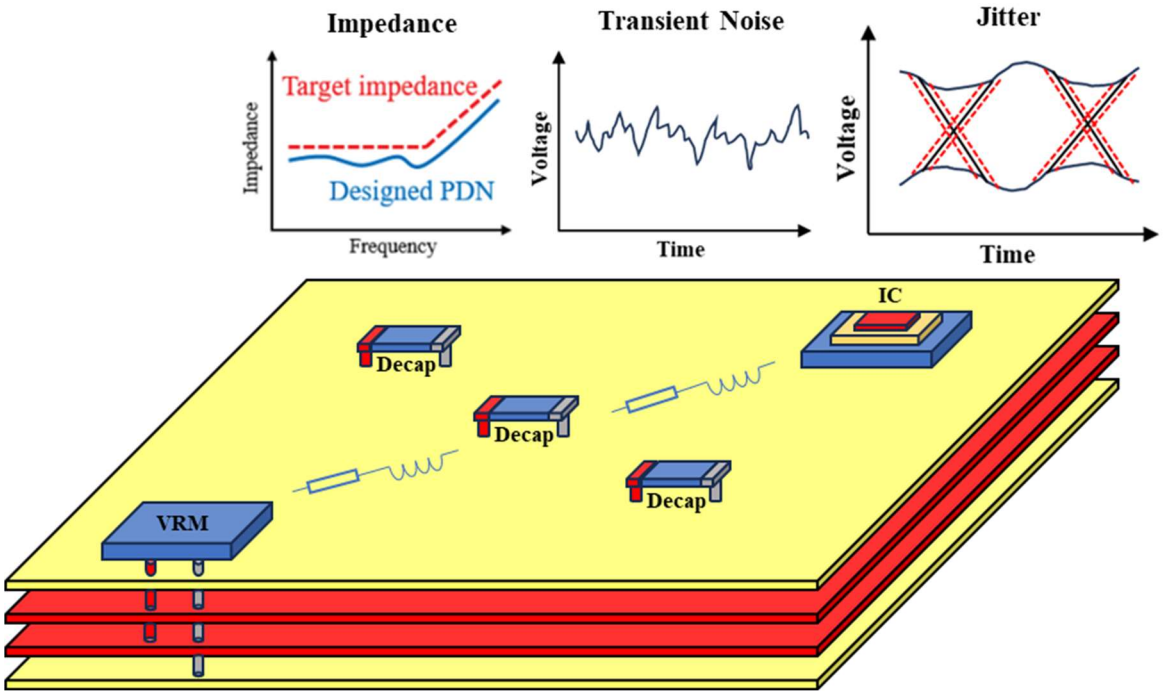



# Power Distribution Network (PDN)

**PDN improves power quality by providing a low impedance transmission path**




**Frequency domain:**

- Parasitic inductance
- Impedance

**Time domain:**

- Transient noise
- PSIJ

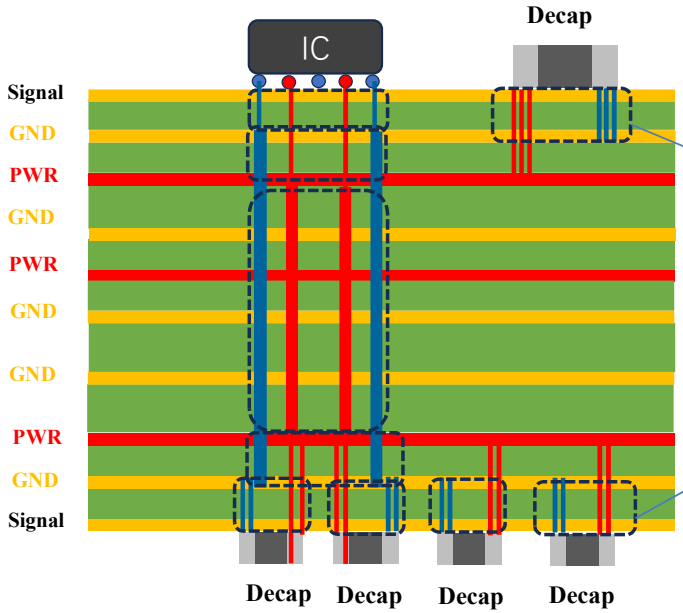
**Design factors:**

- Decoupling capacitor (decap)
- Interconnects (Vias Shapes)

# PDN Structure Modeling

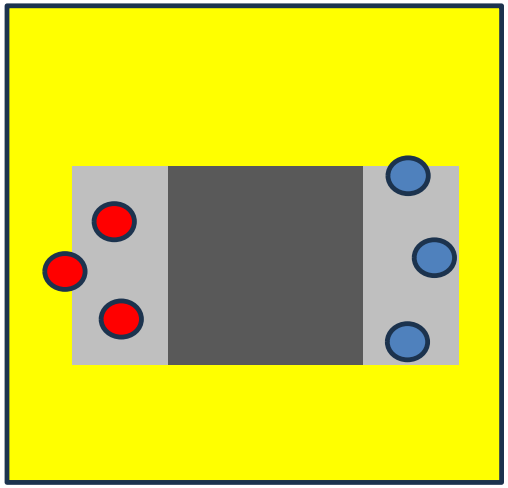
**Modeling and optimization target：**

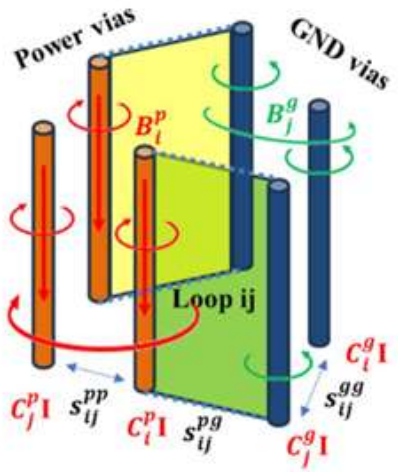
**calculate and reduce the impedance observed from IC ports**




**PDN structure：**

**multilayer planes and vias**



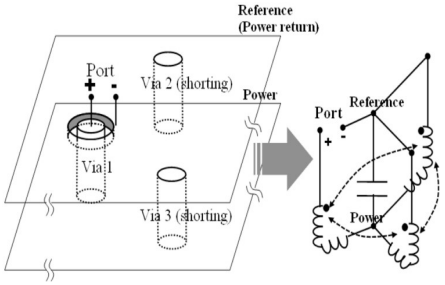



**Main element：**

**Decaps with multiple P/G vias**

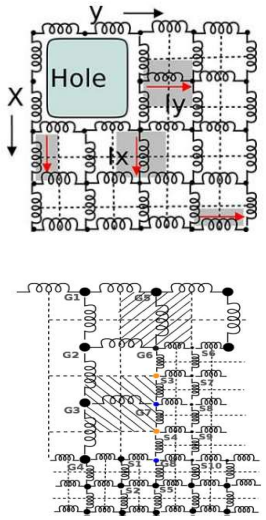
# Traditional Method

## Cavity Model

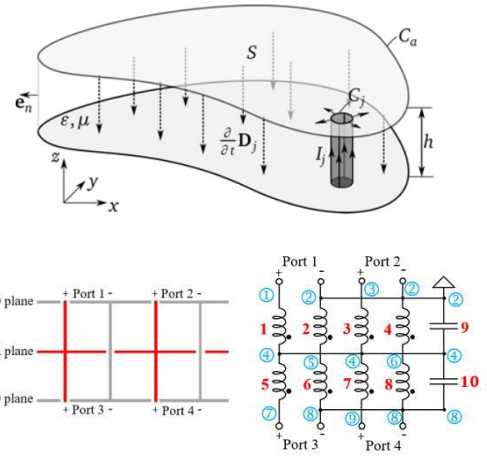



$$Z_{ij}(\omega) = \frac{j\omega\mu d}{ab}\sum_{m=0}\sum_{n=0}\frac{\varepsilon_m^2\varepsilon_n^2}{k_{mn}^2 - k^2} f(x_i, y_i, x_j, y_j)$$

## PEEC Method




## Boundary Element Method

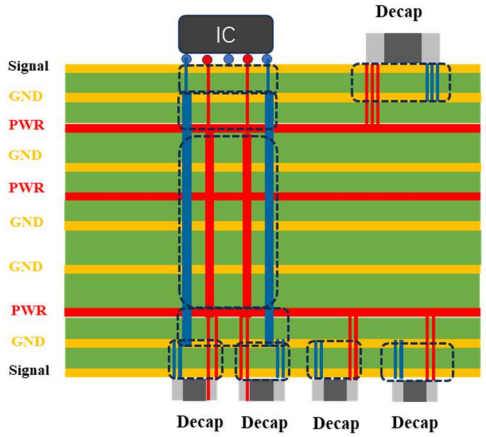



**Main drawbacks：**

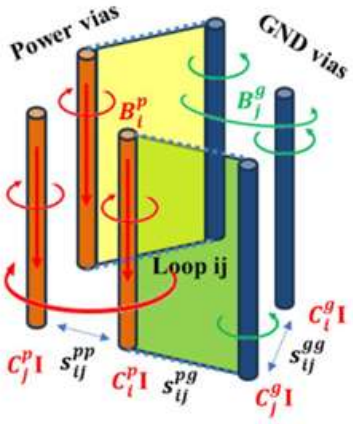
1. Application scenarios are limited
2. Need global re-calculation when structure is changed
3. Not efficient enough for structure optimization

# Proposed Method

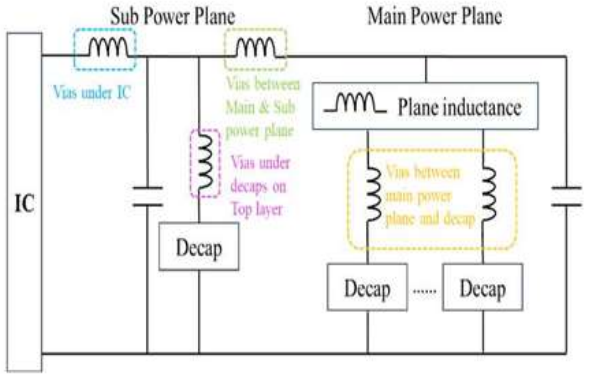
① : Geometry Analysis




② : Pattern Calculation




③ : Circuit Construction



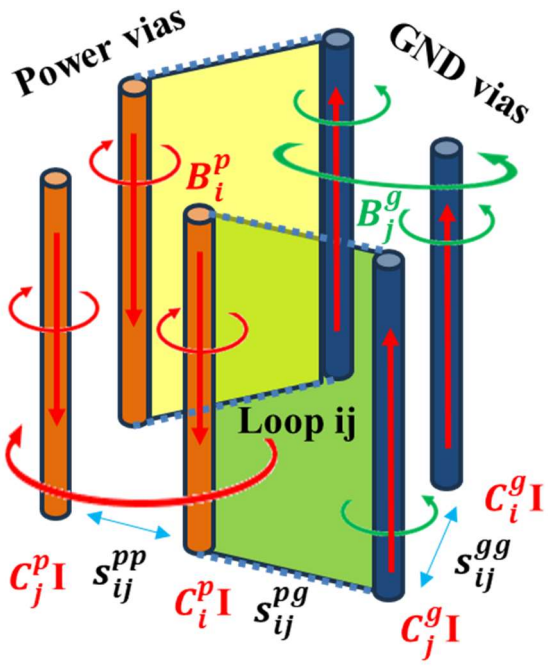

① : The current paths can be divided into several patterns through geometry analysis

② : The inductance of a pattern can be obtained using the proposed calculation method

③ : An equivalent circuit model can be established corresponding to the geometry.

Core innovations: complex pattern and rigorous formulation

# Pattern Inductance Calculation




complex pattern with multiple P/G vias

$C_i^p$ and $C_j^g$: current coefficients of Power/Ground vias

$s_{ij}^{pp}$:distance from power via I to power via j

$s_{ij}^{gg}$:distance from ground via I to ground via j

$s_{ij}^{pg}$:distance from power via I to ground via j

For a current loop *ij* (power via *i* and ground via *j*) From Biot-Savart's Law:

$$\Phi_{ij} = \left(C_i^p + C_j^g\right)\ln\left(\frac{s_{ij}^{pg}}{r}\right) + \sum_{m!=i} C_m^p \ln\left(\frac{s_{mj}^{pg}}{s_{im}^{pp}}\right) + \sum_{n!=j} C_n^g \ln\left(\frac{s_{in}^{pg}}{s_{jn}^{gg}}\right) \quad (1)$$

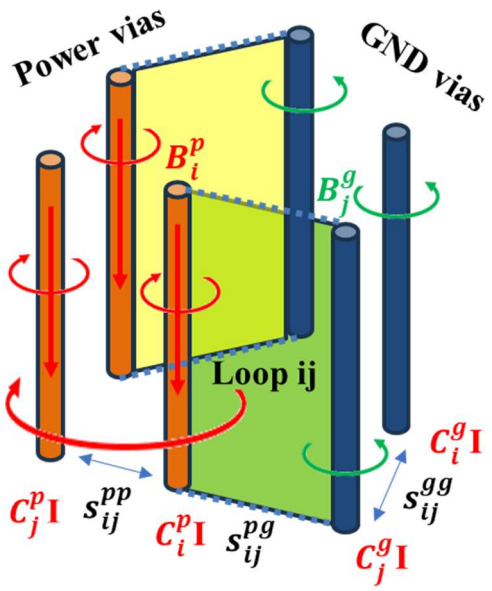
Total inductance is parallel connection of *M*N* inductors:

$$\frac{1}{L_{pul}} = \sum_{i=1}^{M}\sum_{j=1}^{N} \frac{c_i^p c_j^g I}{\Phi_{ij}} \frac{2\pi}{\mu} \sum_{i=1}^{M}\sum_{j=1}^{N} \frac{c_i^p c_j^g}{\left(c_i^p + c_j^g\right) ln\left(\frac{s_{ij}^{pg}}{r}\right) + \sum_{m=1,m\neq i}^{M} c_m^p \, ln\left(\frac{s_{mj}^{pg}}{s_{im}^{pp}}\right) + \sum_{n=1,n\neq j}^{N} C_n^g ln\left(\frac{s_{in}^{pg}}{s_{jn}^{gg}}\right)} \quad (2)$$

**The only unknowns in (2) are *M+N* current coefficients of vias**

# Accurate Current Coefficients

**The only unknowns are *M*+*N* current weights of vias**




Define *M*+*N* magnetic flux of corresponding vias:

$$\Phi_i^p=\sum_j \Phi_{ij}=\frac{\mu I h}{2\pi}\left[c_i^p\sum_j \ln\left(\frac{s_{ij}^{pg}}{r}\right)+\sum_j\left(c_j^g \ln\left(\frac{s_{ij}^{pg}}{r}\right)\right)+\sum_j\sum_{m\neq i} c_m^p \ln\left(\frac{s_{mj}^{pg}}{s_{im}^{pp}}\right)+\sum_j\sum_{n\neq j} c_n^g \ln\left(\frac{s_{in}^{pg}}{s_{jn}^{gg}}\right)\right]$$

$$\Phi_j^g=\sum_i \Phi_{ij}=\frac{\mu I h}{2\pi}\left[c_j^g\sum_i \ln\left(\frac{s_{ij}^{pg}}{r}\right)+\sum_i\left(c_i^p \ln\left(\frac{s_{ij}^{pg}}{r}\right)\right)+\sum_i\sum_{n\neq j} c_n^g \ln\left(\frac{s_{in}^{pg}}{s_{jn}^{gg}}\right)+\sum_i\sum_{m\neq i} c_m^p \ln\left(\frac{s_{mj}^{pg}}{s_{im}^{pp}}\right)\right]$$

The equal-flux condition will result in *M*+*N*-2 equations

$$c_l^p\left(\sum_j \ln\left(\frac{s_{lj}^{pg}}{r}\right)-\sum_j \ln\left(\frac{s_{lj}^{pg}}{s_{il}^{pp}}\right)\right)+c_i^p\left(\sum_j \ln\left(\frac{s_{ij}^{pg}}{s_{li}^{pp}}\right)-\sum_j \ln\left(\frac{s_{ij}^{pg}}{r}\right)\right)+\sum_{m\neq l,i} c_m^p\left(\sum_j \ln\left(\frac{s_{mj}^{pg}}{s_{lm}^{pp}}\right)-\sum_j \ln\left(\frac{s_{mj}^{pg}}{s_{im}^{pp}}\right)\right)$$
$$+\sum_j c_j^g\left(\ln\left(\frac{s_{lj}^{pg}}{r}\right)-\ln\left(\frac{s_{ij}^{pg}}{r}\right)+\sum_{n\neq j}\ln\left(\frac{s_{lj}^{pg}}{s_{nj}^{gg}}\right)-\sum_{n\neq j}\ln\left(\frac{s_{ij}^{pg}}{s_{nj}^{gg}}\right)\right)=0$$

*M*-1 equations

$$c_l^g\left(\sum_i \ln\left(\frac{s_{il}^{pg}}{r}\right)-\sum_i \ln\left(\frac{s_{il}^{pg}}{s_{lj}^{gg}}\right)\right)+c_j^g\left(\sum_i \ln\left(\frac{s_{ij}^{pg}}{s_{lj}^{gg}}\right)-\sum_i \ln\left(\frac{s_{ij}^{pg}}{r}\right)\right)+\sum_{n\neq l,j} c_n^g\left(\sum_i \ln\left(\frac{s_{in}^{pg}}{s_{ln}^{gg}}\right)-\sum_i \ln\left(\frac{s_{in}^{pg}}{s_{jn}^{gg}}\right)\right)$$
$$+\sum_i c_i^p\left(\ln\left(\frac{s_{il}^{pg}}{r}\right)-\ln\left(\frac{s_{ij}^{pg}}{r}\right)+\sum_{m\neq i}\ln\left(\frac{s_{lj}^{pg}}{s_{im}^{pp}}\right)-\sum_{m\neq i}\ln\left(\frac{s_{ij}^{pg}}{s_{im}^{pp}}\right)\right)=0$$

*N*-1 equations

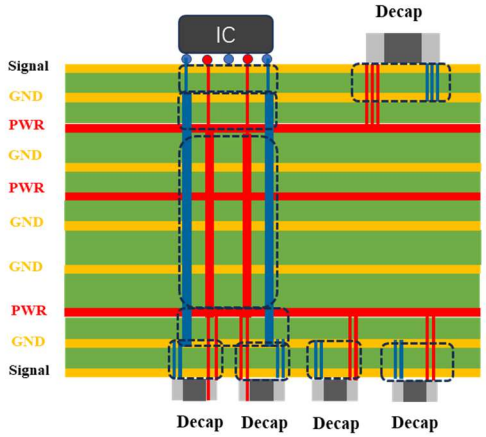
$$c_1^p+c_2^p+\cdots+c_M^p=1$$
$$c_1^g+c_2^g+\cdots+c_N^g=1$$

2 equations
(Current conservation)

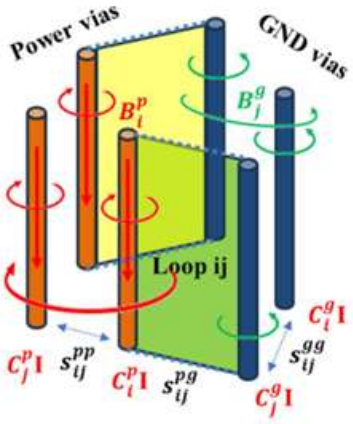
Linear system of *M*+*N* equations for solving *M*+*N* unknown current coefficients

# Proposed Method

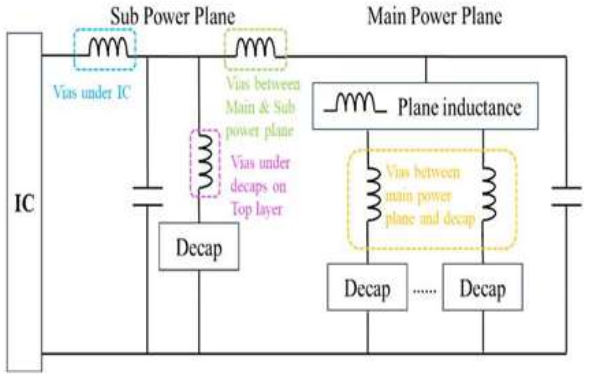
① : Geometry Analysis




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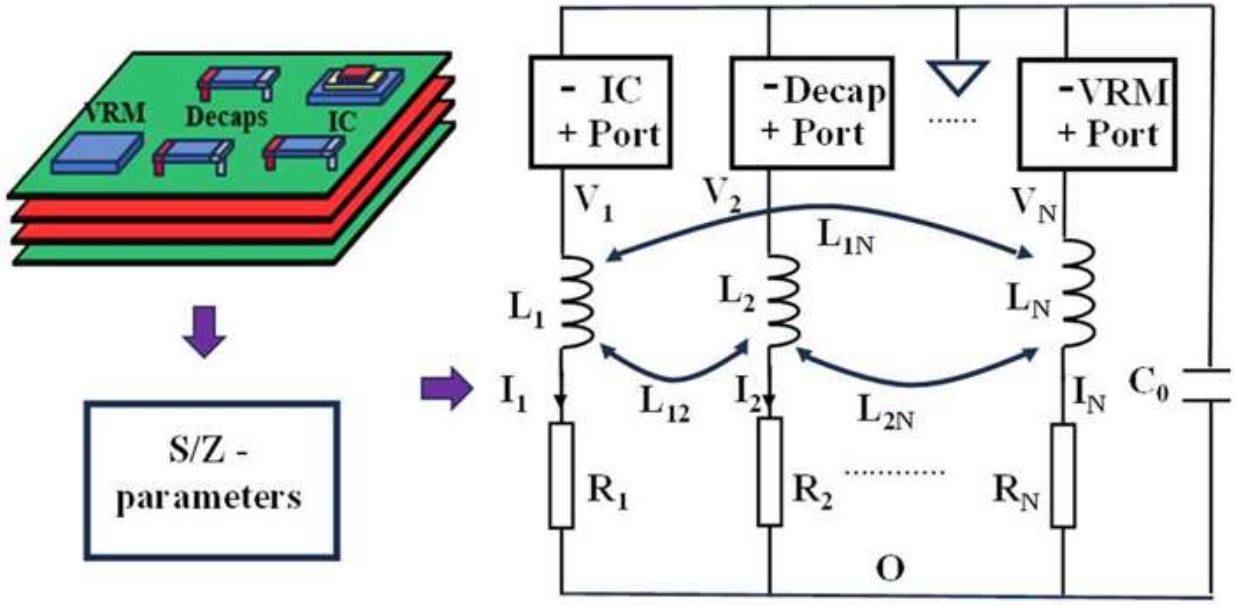

① : The current paths can be divided into several patterns through geometry analysis

② : The inductance of a pattern can be obtained using the proposed calculation method

③ : An equivalent circuit model can be established corresponding to the geometry.

Core innovations: complex pattern and rigorous formulation

# Equivalent Circuit Model



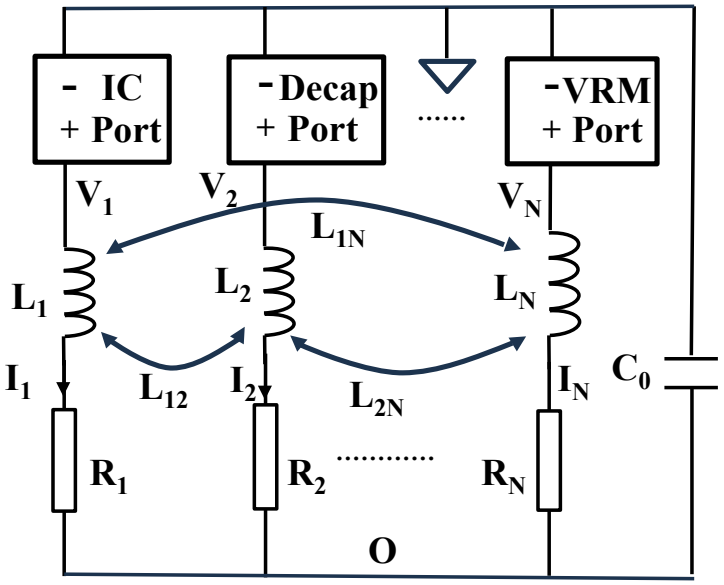

1.Ports are positioned at critical lumped elements including IC, Decaps, VRM

2.L-R connected in series for each port and mutual inductance for coupling

3.$C_0$ stands for the inter-plane capacitance, which can be regarded as an open circuit

4.Special property: loop inductance determines the impedance

# Special Property of Circuit Model



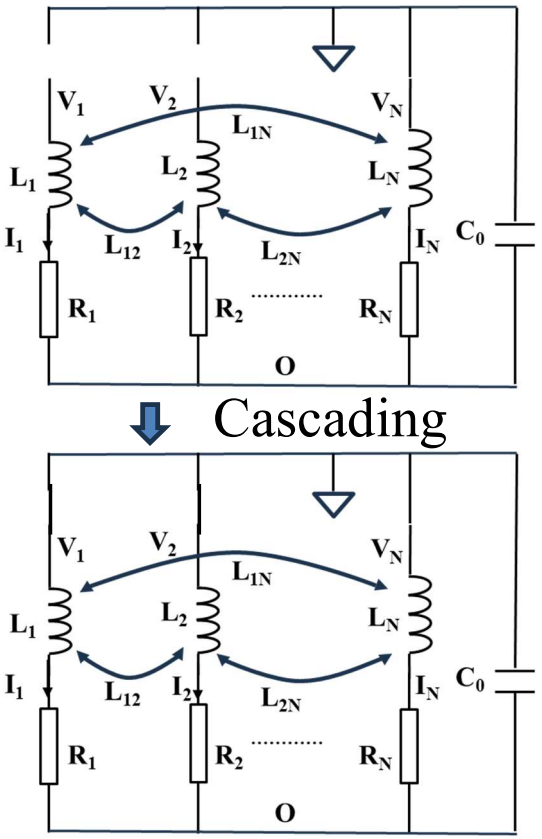

$\boldsymbol{V} = [V_1, V_2 \ldots, V_N]^T$ and $\boldsymbol{I} = [I_1, I_2, \ldots, I_N]^T$

Port voltage and current vector

$$\boldsymbol{V} - V_o = \begin{bmatrix} V_1 - V_o \\ V_2 - V_o \\ \cdots \\ V_N - V_o \end{bmatrix} = \begin{bmatrix} Z_{11}^b & Z_{12}^b & \cdots & Z_{1N}^b \\ Z_{21}^b & Z_{22}^b & \cdots & Z_{2N}^b \\ \cdots & \cdots & \cdots & \cdots \\ Z_{N1}^b & Z_{N2}^b & \cdots & Z_{NN}^b \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdots \\ I_N \end{bmatrix} = \boldsymbol{Z^b I}.$$

The element of $\boldsymbol{Z^b}$: $Z_{ii}^b = R_i + j\omega L_i$, and $Z_{ij}^b = j\omega L_{ij}$.

# Special Property of Circuit Model




$$V_i = \sum_{j=1}^{N-1}[(Z_{ij}^b - Z_{Nj}^b) - (Z_{iN}^b - Z_{NN}^{new})]I_j, i = 1,2,...,N-1$$

Define loop inductance between port $i$ and $j$ :

$$L_{ij}^{loop} = L_i + L_j - 2L_{ij}$$

Diagonal: $$Z_{ii} = Z_{ii}^b + Z_{NN}^{new} - 2Z_{iN}^b = R_i + R_N + Z_N^{decap} + j\omega L_{iN}^{loop}$$

Non-diagonal: $$Z_{ij} = Z_{ij}^b - Z_{iN}^b - Z_{jN}^b + Z_{NN}^{new}$$

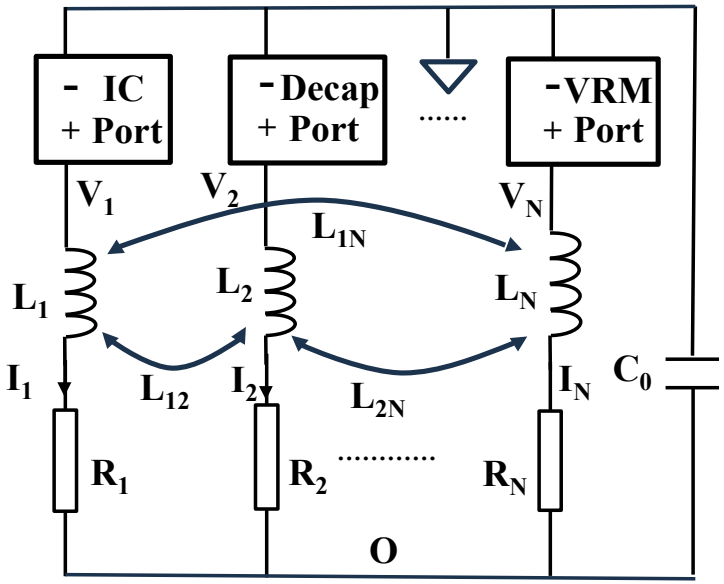
$$= \frac{1}{2} j\omega(L_{ij}^{loop} - L_{iN}^{loop} - L_{jN}^{loop}) + R_N + Z_N^{decap}$$

③

The ($N$-1)×($N$-1) matrix is determined by all $L^{loop}$ !
When more than one element is connected:
Cascade the ($N$-1)×($N$-1) matrix with other elements

**The PDN impedance is determined by all $L^{loop}$ !**

# Circuit Model Conversion




Unknowns: $N$ resistance and $N+C_N^2$ inductance

Constructing $C_N^2$ independent current paths:

$$Z_{msn} = R_m + R_n + j\omega L_{mn}^{loop}$$

$Z_{msn}$: Impedance of port $m$ after shorting the port $n$

Parameters initialization:

$$f_m^r = \frac{1}{2\pi\sqrt{L_m^{initial} C_0}} \qquad L_{mn}^{initial} = -\frac{1}{2}\left(L_{mn}^{loop} - L_m^{initial} - L_n^{initial}\right)$$

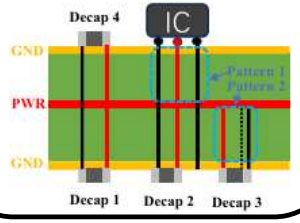
Least square optimization:

$$\min_{L_{self}, L_{mutual}, R} \sum_{m=1}^{N} \sum_{n=m+1}^{N} \| Z_{msn}^{circuit} - Z_{msn}^{actual} \|_2^2$$

**Result: $L_i$ *and* $L_{ij}$ are non-unique, but $L^{loop}$ is unique!**

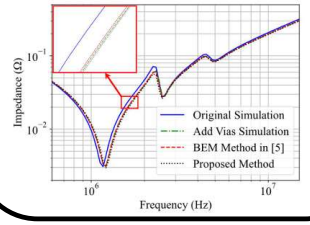
# Equivalent Circuit Fine-tuning

Structural modification

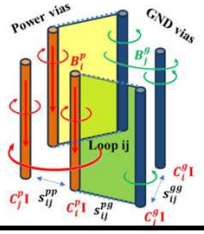



Fine-tuning

Altered PDN Impedance

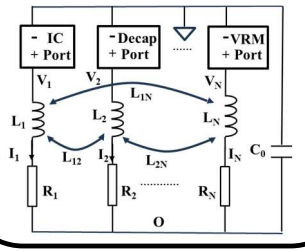



Local Re-modeling




$$L_{ij}^{new} = L_{ij}^{old} - \frac{1}{2}\Delta L_{ij}^{loop}$$

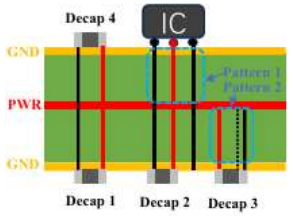
Fine-tuned Circuit




**Fine-tuning foundation: loop inductances determine the PDN impedance**

# Overall Framework

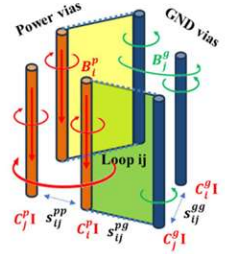
**PDN structure**




**Pattern**

**Inductance**

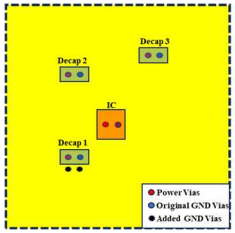
**Complex Pattern**




**Local**

**Modeling**

**PDN fine-tuning**



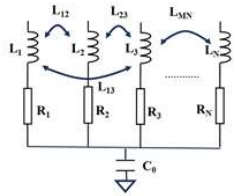

**Full-wave simulation**

**PDN S-parameters**

**Circuit**

**Transform**

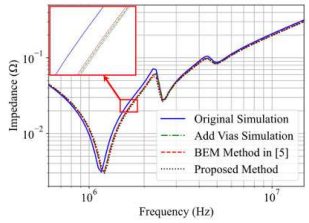
**Equivalent circuit model**



**Circuit**

**Solver**

**Tuning result**



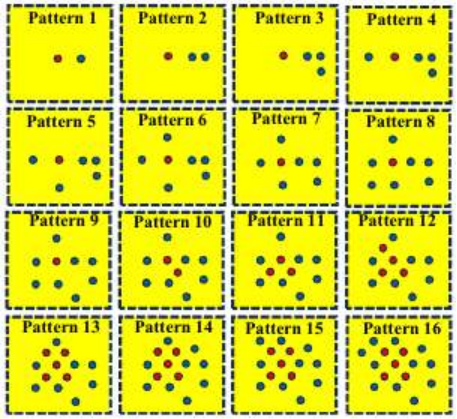

**Main advantages：Fast local re-modeling by updating and solving equivalent circuit model**

# Method Validation

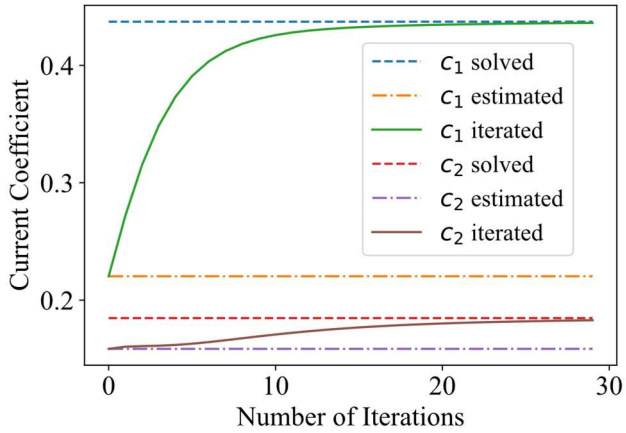
**Pattern-based method[1]** **VS** **proposed method**

- **Simple patterns** with 1 power and N ground vias
- **Empirical formula** for current weights estimation
- **Not applicable** for fine-tuning scenario

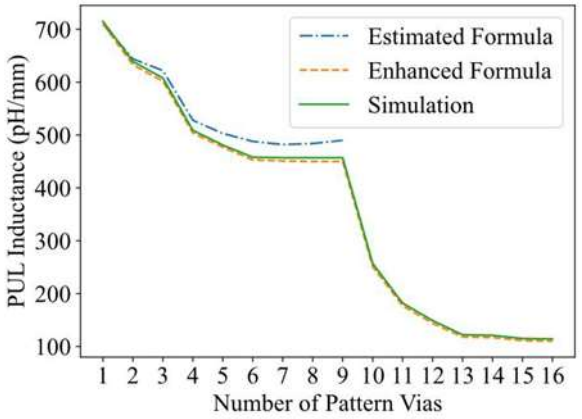
- **Complex patterns** with M power and N ground vias
- **Rigorous solution** for current weights determination
- **Fine-tuning modeling** based on equivalent circuit




**Patterns for comparison**




**Current weights for pattern 7**

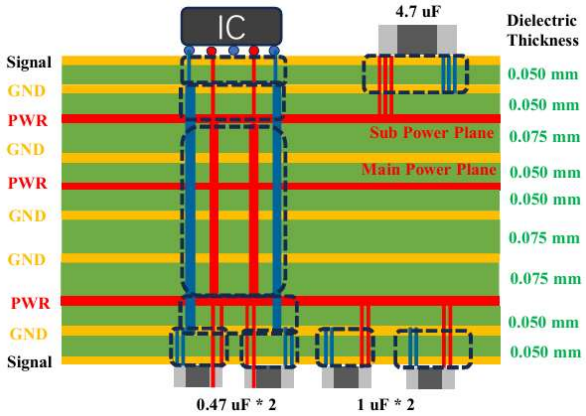



**Inductance for pattern 1-16**

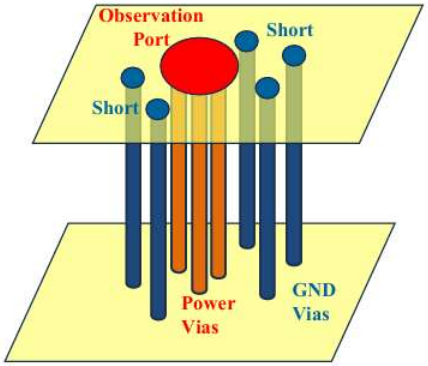

[1] J. Sun, H. Wang, K. Wu and J. Fan, "A Pattern-Based Analytical Method for Impedance Calculation of the Power Distribution Network in Mobile Platforms," in *IEEE Transactions on Electromagnetic Compatibility*, vol. 63, no. 3, pp. 912-921, June 2021

# Method Validation

## PDN board geometry




## Pattern Simulation Config.




## Equivalent Circuit Model

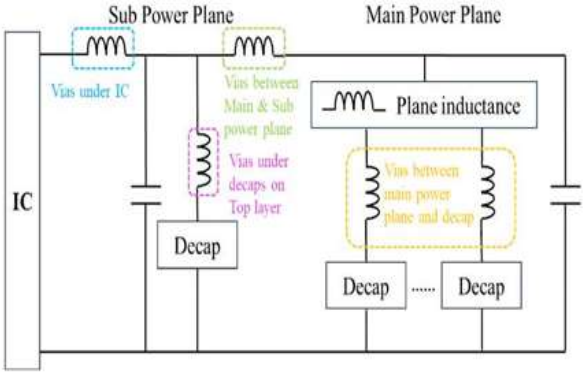
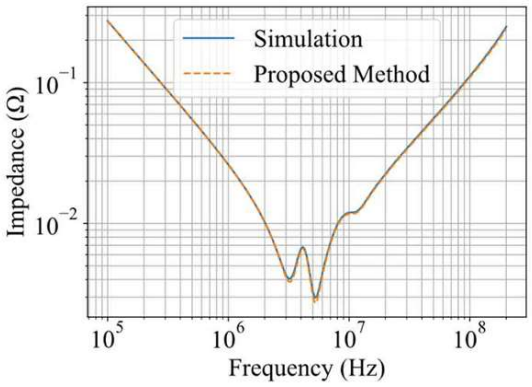



Table I. Inductance of different complex patterns obtained by the proposed method and the commercial simulation tool.

| Methods / Inductors | Proposed Method | Simulation |
|---|---|---|
| Under IC | 63 pH | 65 pH |
| Between Main & Sub power Plane | 47 pH | 48 pH |
| Decap 1 Branch | 16 pH | 16 pH |
| Decap 2 Branch | 114 pH | 112 pH |
| Decap 3 Branch | 107 pH | 104 pH |
| Decap 4 Branch | 106 pH | 109 pH |
| Decap 5 Branch | 111 pH | 108 pH |




| Method | Time |
|---|---|
| Simulation | 43 s |
| Proposed Method | ~0.1 s |

**Region-By-Region Comparison**

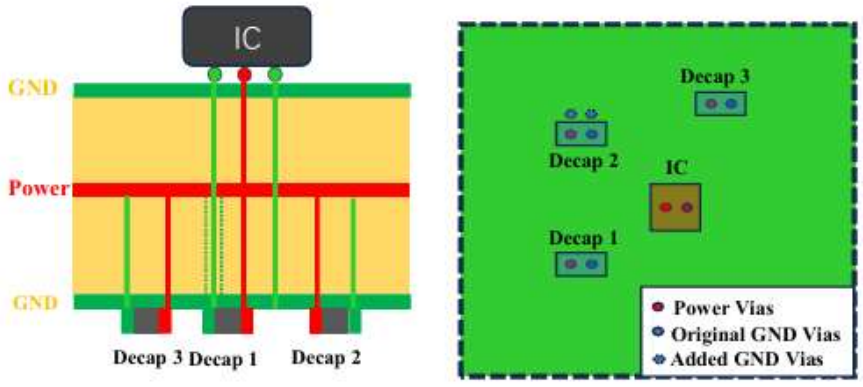
**Impedance Validation**

**Time Comparison**

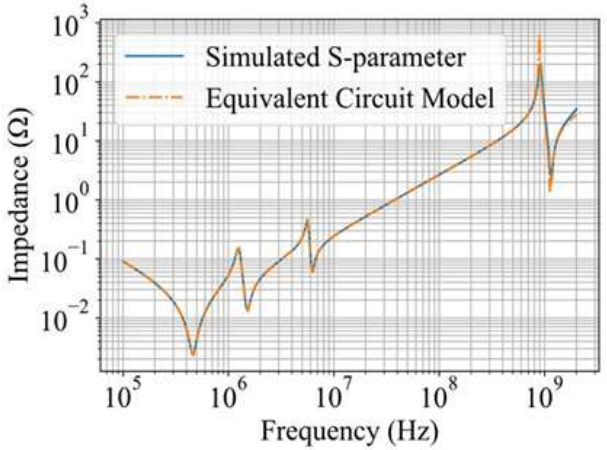
# Method Validation

## Validation of the Equivalent Circuit Conversion Method

PDN Geometry → (Simulation) → S/Z-parameters → (Conversion) → Circuit Model → (Solve) → PDN Impedance




PDN Geometry For Validation




Comparison of PDN Impedance

**Highly consistent with simulation (maximum discrepancy of about 5%)**

# Method Validation

## Validation of the Equivalent Circuit Fine-tuning Method

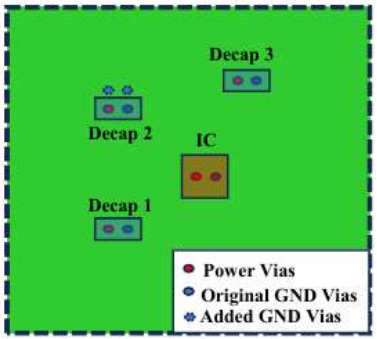
**Add ground vias**

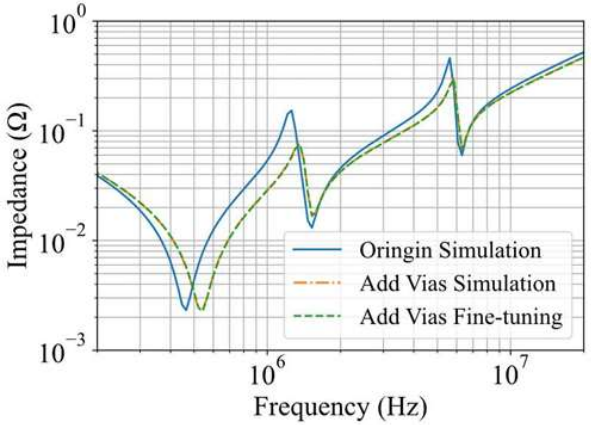
**Impedance Result:**

- **Resonance frequency**
- **Resonance peak and valley**
- **Max 5% error compared to Sim**

**Time Efficiency：**

- **Analytical expression**
- **~1000 times faster than BEM method**
- **>10000 times faster than simulation**




| | Modeling time |
|---|---|
| Simulation | 13 s |
| BEM* | ~ 1 s |
| Proposed Method | ~ 0.002 s |

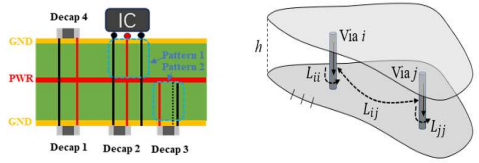
**High time efficiency while maintaining modeling precision ！**

*"Efficient DC and AC Impedance Calculation for Arbitrary-Shape and Multilayer PDN Using Boundary Integration," IEEE Trans. Signal Power Integrity, vol. 1, pp. 1-11, 2022.

# Work Overview of Our Team

## Our Previous Works ①②③④⑤

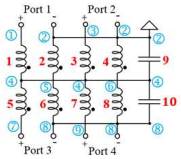
### Structure Modeling

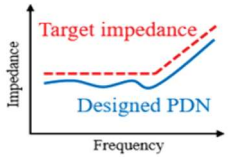




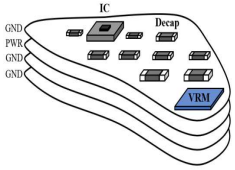


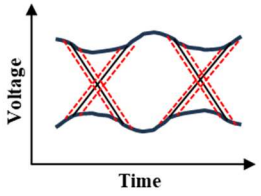
### Impedance calculation



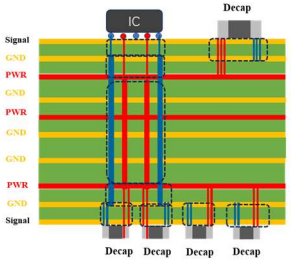

### Decap Layout Optimization

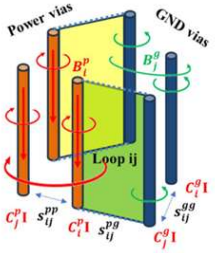


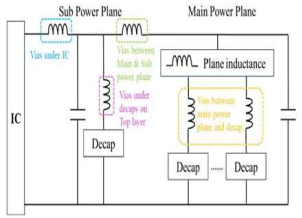




## PDN Fine-tuning Design Framework

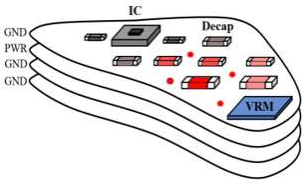
### Improved Pattern-Based Inductance Calculation

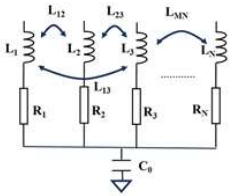


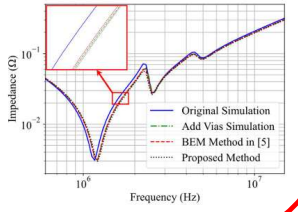




### Equivalent Circuit Fine-tuning





①"Efficient DC and AC Impedance Calculation for Arbitrary-Shape and Multilayer PDN Using Boundary Integration," in *IEEE Transactions on Signal and Power Integrity*, vol. 1, pp. 1-11, 2022

②"Fast impedance prediction for power distribution network using deep learning," *Int. J. Numer. Model.*, vol. 35, no. 2, p. e2956, 2022

③"Decoupling Optimization for Complex PDN Structures Using Deep Reinforcement Learning," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 71, no. 9, pp. 3773-3783, Sept. 2023

④"A Novel Physics-Assisted Genetic Algorithm for Decoupling Capacitor Optimization," in *IEEE Transactions on Microwave Theory and Techniques*, vol. 72, no. 8, pp. 4498-4507, Aug. 2024

⑤"PDN Noise-Jitter Co-Optimization Using Physics-Assisted Genetic Algorithm," *2024 IEEE International Symposium on (EMC+SIPI)*, Phoenix, AZ, USA, 2024, pp. 526-531

Zhejiang University

# Thanks for Listening!

Presenter: Li Jiang

Advisors: Er-ping Li, Ling Zhang

Hybrid IBIS Summit at IEEE WAI 2026
Hangzhou, China
November 6, 2026

College of Information Science and Electronic Engineering