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A Phase-Noise-Based Method for Calculating Random Jitter Amplification in Clock Channels

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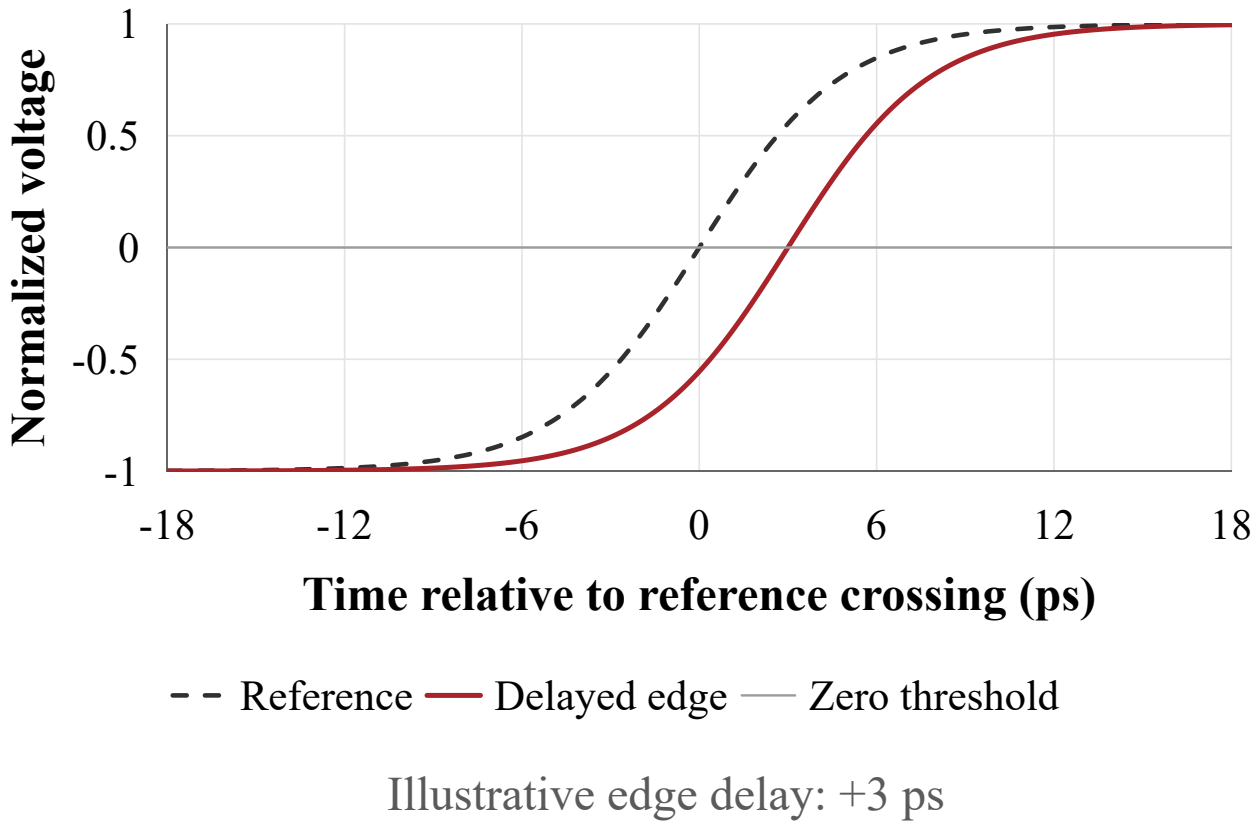
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Outline

- Background
- Phase-Noise-Based Calculation of RJAF
- Validation and Frequency-Domain Analysis
- Conclusions and Future Work

Timing Jitter in Clock Channels

- Clock edges provide the timing reference for sampling and synchronization.
- Timing jitter describes deviations from reference transition times.
- Random TIE RMS quantifies timing uncertainty.

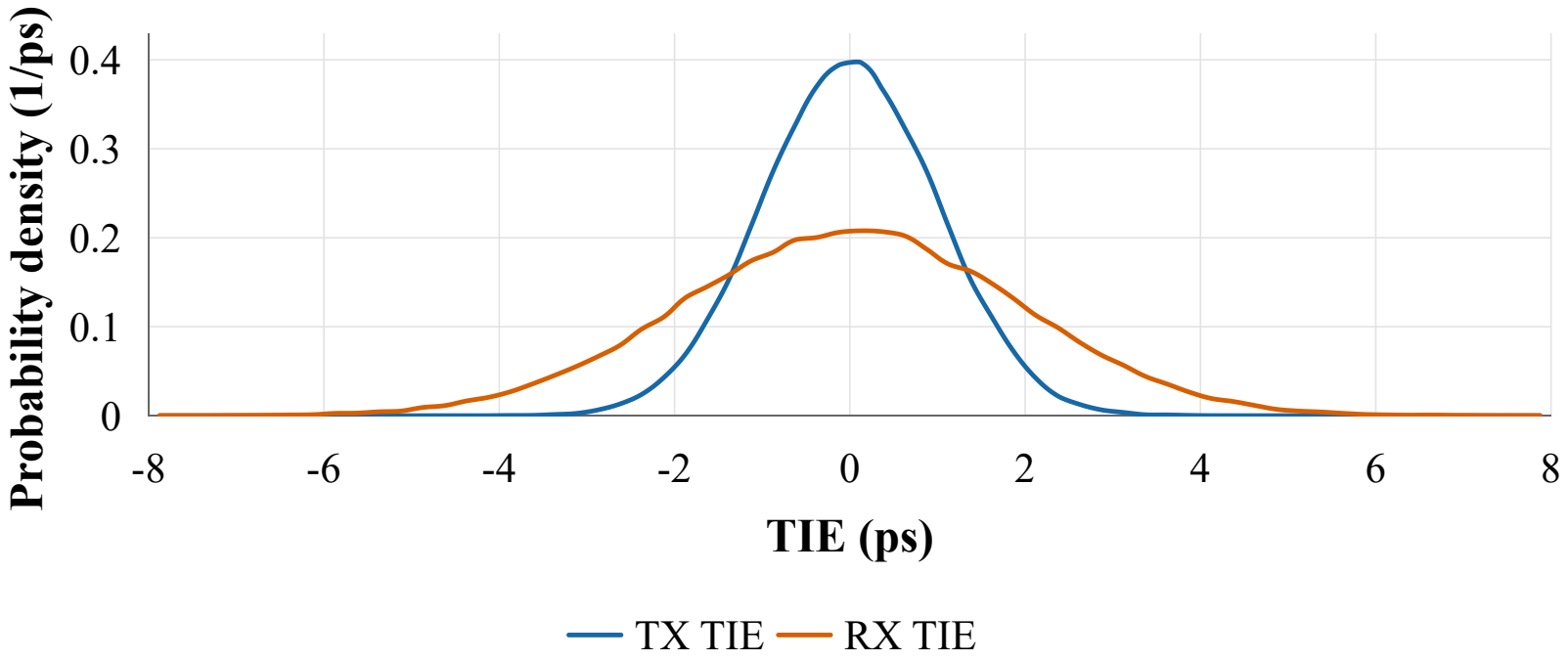


Illustrative edge delay: +3 ps

- TIE = actual crossing time − reference crossing time

Jitter Amplification in Clock Channels

- A linear channel model can change existing timing jitter without an added voltage-noise source.



DDR channel
Clock: 3.4 GHz
Jitter bandwidth: 3.4 GHz

TX RMS ≈ 1 ps
RX RMS ≈ 1.92 ps

RJAF ≈ 1.92

RJAF = RX TIE RMS / TX TIE RMS

RJAF > 1:amplification RJAF < 1:attenuation

What Determines Jitter Amplification?

- **Physical mechanism**

  Why can timing jitter grow without an added noise source?

- **Channel and clock frequency**

  Why can changing the clock frequency alter RJAF in the same channel?

- **Input jitter spectrum**

  Why can equal input RMS jitter produce different output jitter?

- A frequency-domain model links the channel response and input jitter spectrum to the output zero-crossing timing error.

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Phase Modulation and Sidebands

Jitter-free carrier

$$x(t) = A\cos(2\pi f_c t)$$

$$\phi(t) = -2\pi f_c \tau(t)$$

The same carrier with timing jitter

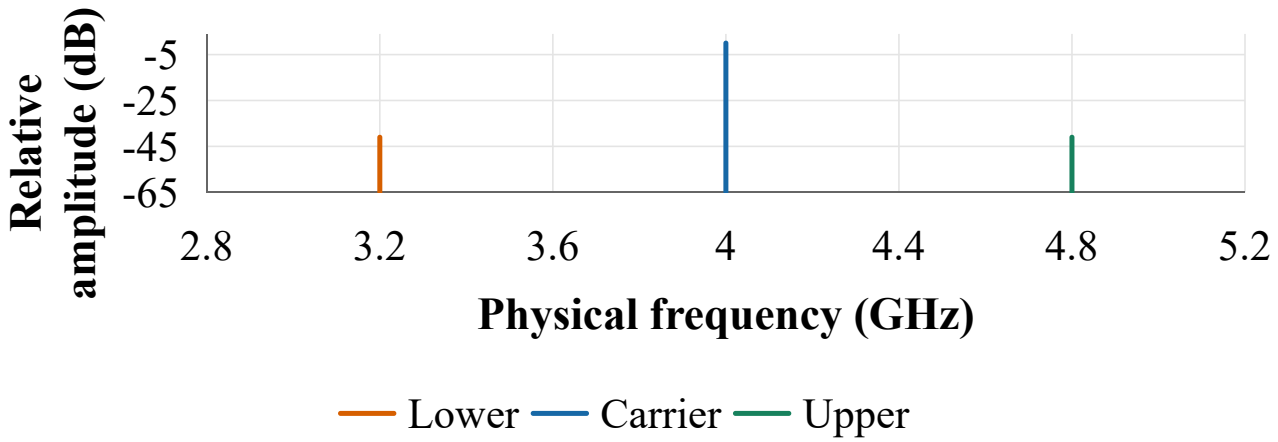
$$x_j(t) = x[t - \tau(t)] = A\cos[2\pi f_c t + \phi(t)]$$

$\tau(t)$: input time shift; positive means delayed.
The phase perturbation belongs to this carrier.

Small-phase expansion

$$x_j(t) \simeq \frac{A}{2}e^{j2\pi f_c t} + \frac{A}{2}j\phi(t)e^{j2\pi f_c t} + \text{c.c.}$$

c.c.: complex conjugate



One sinusoidal timing component

$$\tau(t) = \tau_p \cos(2\pi f_j t)$$

$f_c = 4$ GHz; $f_j = 0.8$ GHz
Sidebands: $f_c - f_j$ and $f_c + f_j$

f_j: jitter frequency (offset from carrier).
SJ = 0.5 ps RMS; first-order illustration.

- Timing jitter produces phase modulation and correlated sidebands.

Single-Tone Jitter Transfer

$$G_{\sin}(f_j) = \frac{1}{2}\left[\frac{H(f_c+f_j)}{H(f_c)} + \frac{H(f_j-f_c)}{H(-f_c)}\right]$$

$|G_{\sin}(f_j)|$: output-to-input timing amplitude ratio at jitter frequency f_j

- Normalize sideband responses to the carrier.
- Keep complex phase when combining contributions.

Second term: conjugate lower-sideband response

$$|a+b|^2 = |a|^2 + |b|^2 + 2\mathrm{Re}\{ab^*\}$$

Generic complex sum: $|a| = 1$, $|b| = 0.7$

In phase  2.89

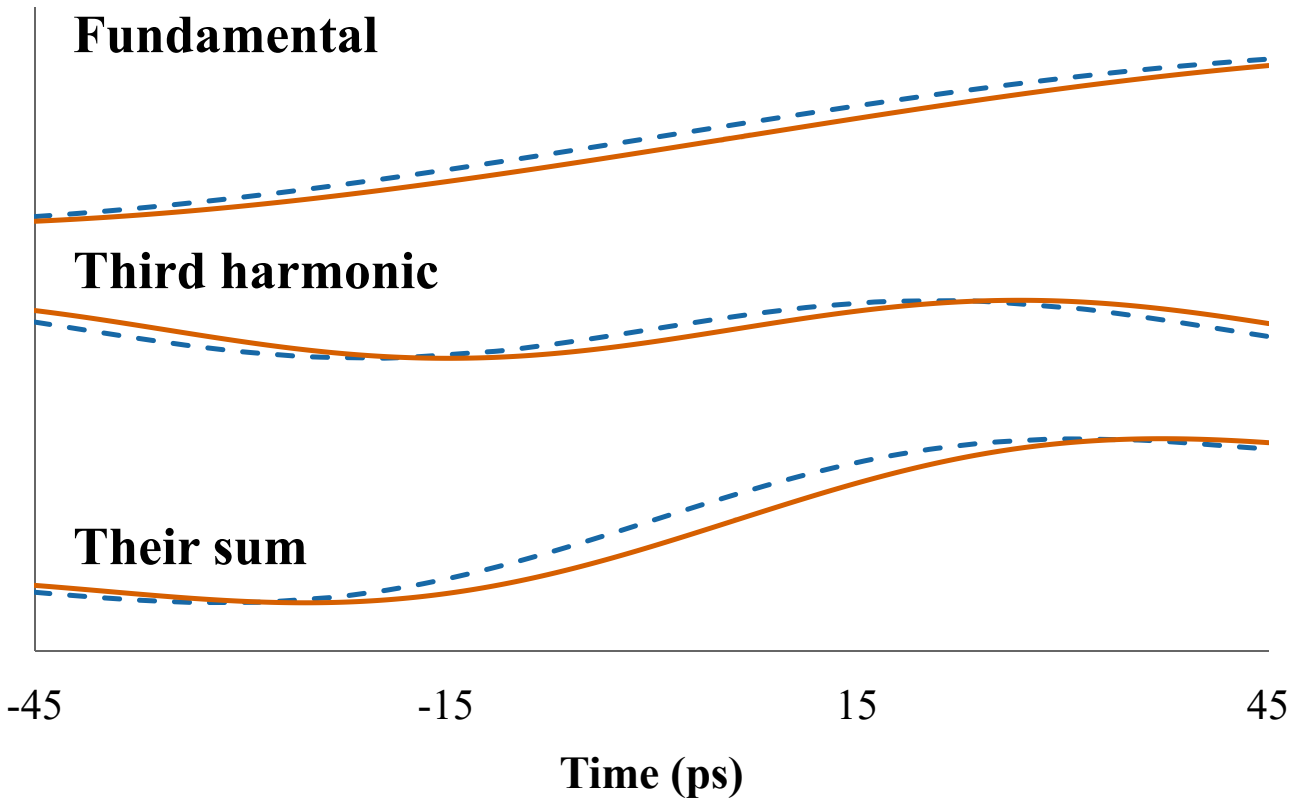
Opposite phase  0.09

Power-only sum: 1.49. Values illustrate $|a + b|^2$.
This arithmetic example is not a channel RJAF.

- Relative phase changes the coherent sum and the timing response.

[1] Rao F, Hindi S., "Mechanism of jitter amplification in clock channels," in Proc. DesignCon, Santa Clara, CA, USA, Jan. 2014, pp. 1-24

Time Disturbance Across Clock Harmonics



$$x(t) = \sum_{k \in \mathcal{H}} c_k e^{j2\pi k f_c t}$$

c_k: TX Fourier coefficient.
Include positive and negative k.

$$\phi_k(t) = -2\pi k f_c \tau(t)$$

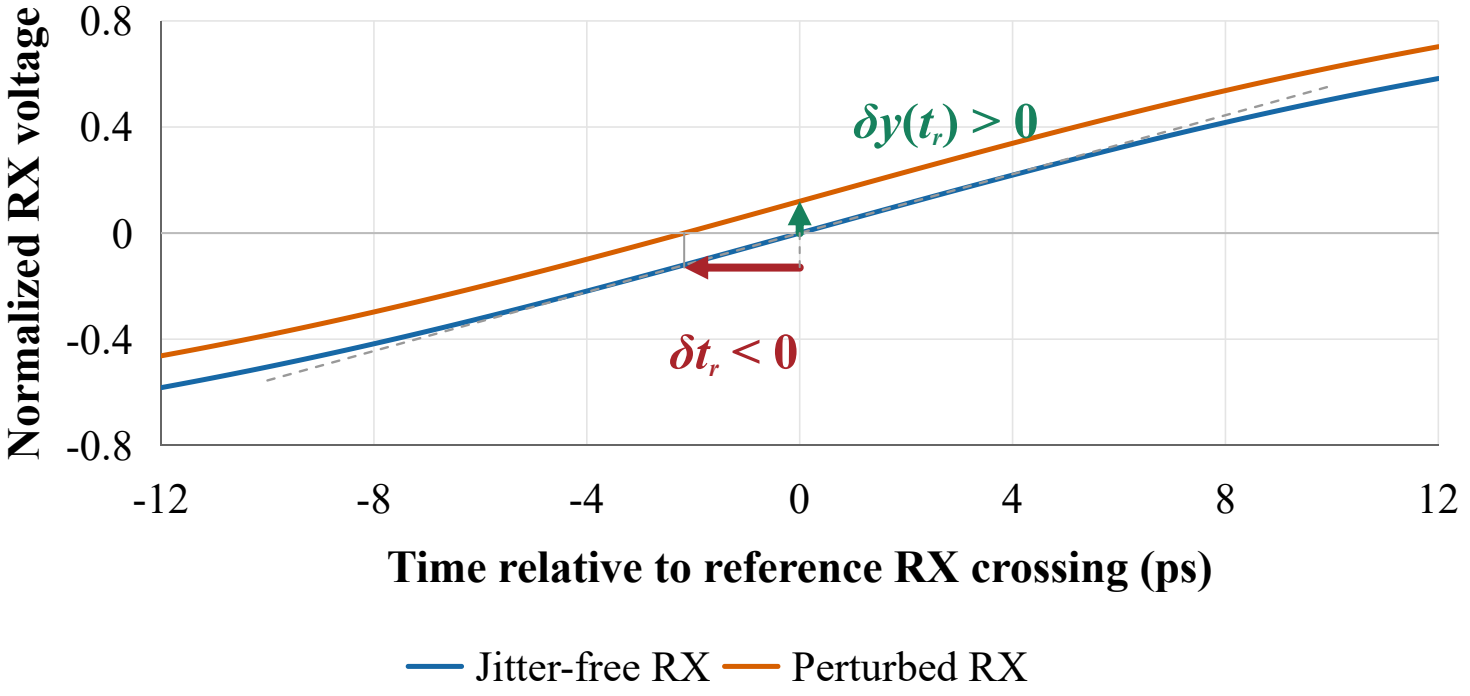
$$x_j(t) \simeq x(t) - \tau(t)x'(t)$$

Same time displacement
phase change scales with k

f_c = 4 GHz; common shift = 6 ps.
Vertically offset traces; conceptual illustration.

- A common time disturbance drives all harmonic sidebands coherently.

Voltage Perturbation and Crossing Time



$y_0(t)$: jitter-free RX waveform

t_r: reference crossing time
r: rising or falling edge

$\delta y(t_r)$: local voltage change

K_r: signed local slope

$$\delta t_r \simeq -\frac{\delta y(t_r)}{K_r}, \qquad K_r = y_0'(t_r)$$

Local first-order geometry; the voltage change represents the combined RX timing disturbance.

- Use the same complete RX waveform for the perturbation and slope.

Multi-Harmonic Jitter Transfer

1. First-order TX perturbation for a single jitter component at f_j

$$\tau_{f_j}(t) = \hat{\tau} e^{j2\pi f_j t}, \qquad \delta x_{f_j}(t) = -\tau_{f_j}(t) x'(t)$$

2. RX voltage perturbation at reference crossing t_r through $H(f)$

$$\delta y_{f_j}(t_r) = -\hat{\tau} e^{j2\pi f_j t_r} \sum\nolimits_{k\in\mathcal{H}} j2\pi k f_c c_k H(kf_c + f_j) e^{j2\pi k f_c t_r}$$

3. Multi-harmonic transfer function $G_r(f_j)$ via local slope $\delta t_r = -\delta y(t_r)/K_r$

$$G_r(f_j) = \frac{\sum_{k\in\mathcal{H}} j2\pi k f_c c_k H(kf_c + f_j) e^{j2\pi k f_c t_r}}{K_r}$$

$$\delta t_{r,f_j} = \hat{\tau} G_r(f_j) e^{j2\pi f_j t_r}$$

Single-carrier limit: retain only $k = \pm 1$ and use its RX crossing and slope

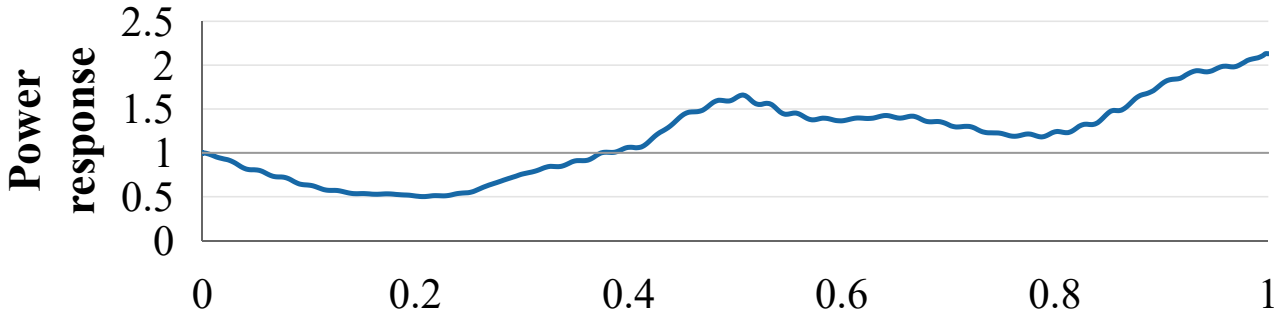
$$G_r(f_j)\big|_{k=\pm 1} = G_{\sin}(f_j)$$

- The single-carrier response is a special case of the multi-harmonic response.

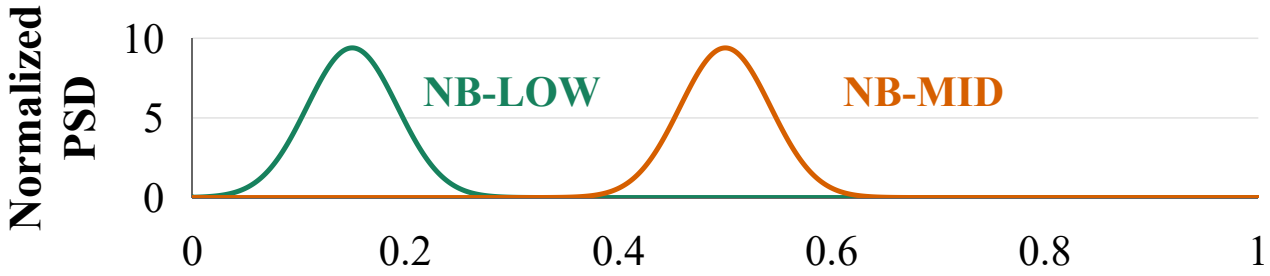
Random Jitter and Spectral Weighting

$$q_r(f_j) = |G_r(f_j)|^2$$

DDR channel, f_c = 4 GHz; selected crossing *r*



Both horizontal axes: f_j/f_c



Input spectra have equal area in the normalized frequency coordinate.

- RJAF² is the input-PSD-weighted average of $|G_r|^2$.

TX variance

$$\sigma^2_{j,\mathrm{TX}} = \int_0^{B_j} S_\tau(f_j)\, df_j$$

RX variance at crossing r

$$\sigma^2_{j,\mathrm{RX},r} = \int_0^{B_j} q_r(f_j) S_\tau(f_j)\, df_j$$

Normalize by the input variance

$$\mathrm{RJAF}_r^2 = \frac{\sigma^2_{j,\mathrm{RX},r}}{\sigma^2_{j,\mathrm{TX}}} = \frac{\int_0^{B_j} q_r(f_j) S_\tau(f_j)\, df_j}{\int_0^{B_j} S_\tau(f_j)\, df_j}$$

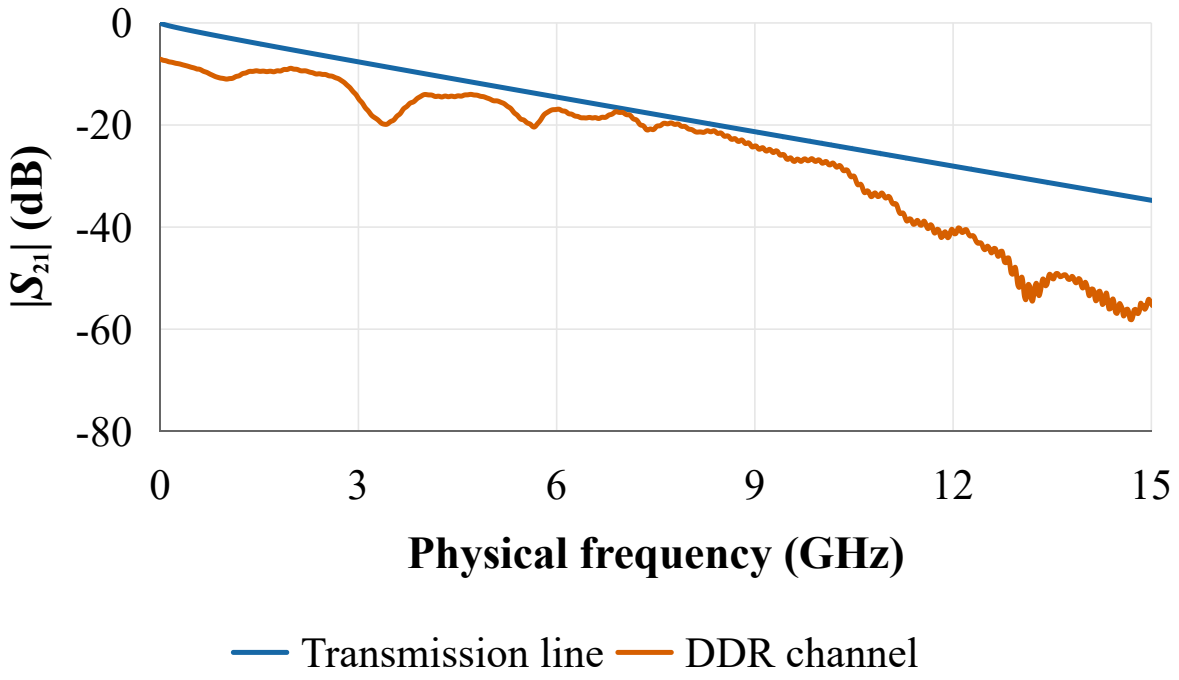
S_τ: one-sided timing PSD (s²/Hz).
B_j: input jitter bandwidth; zero-mean jitter.

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Simulation and Calculation Conditions

Measured channel responses

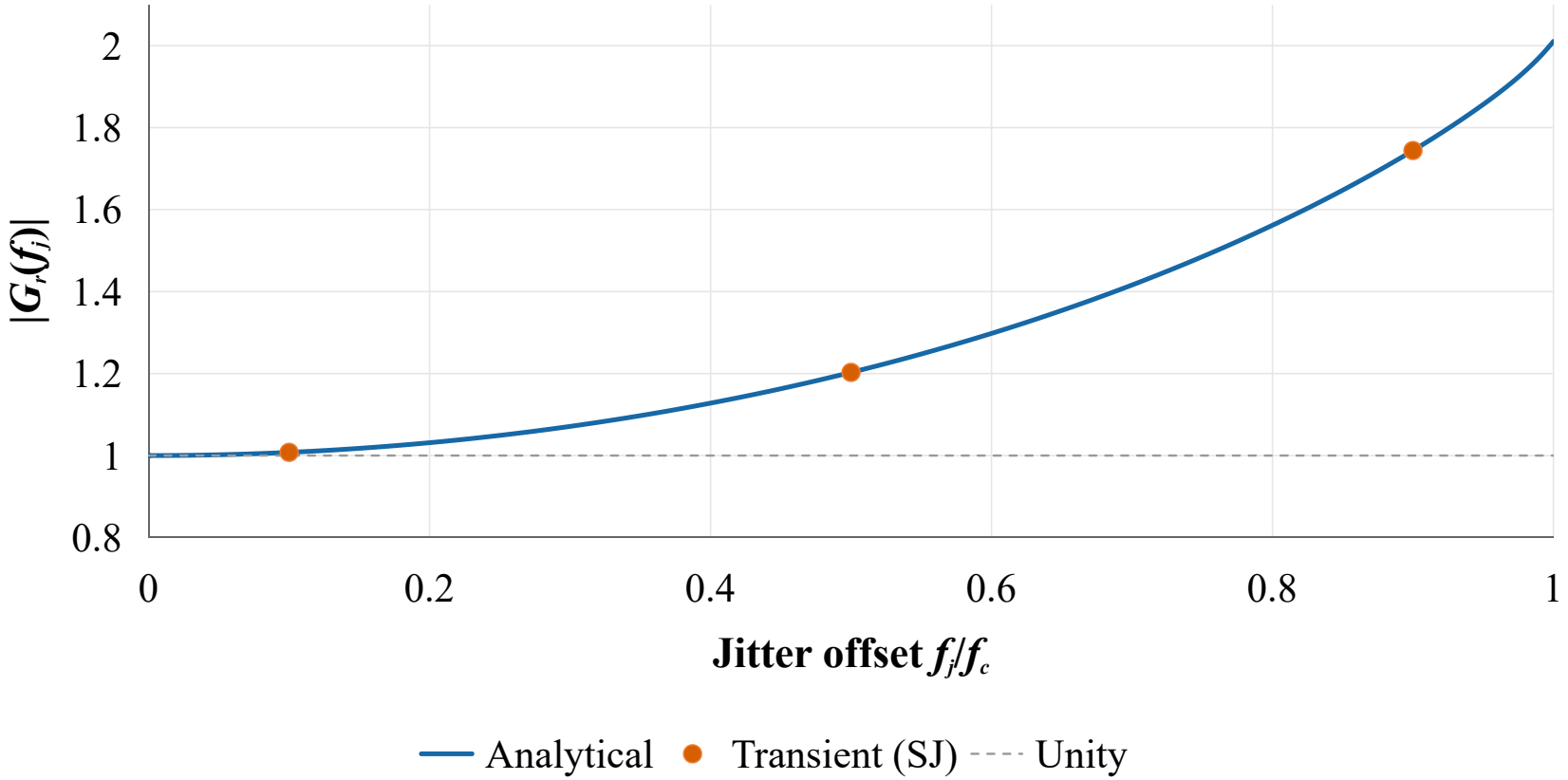


	Common conditions
Excitation	10 ps rising/falling edge RJ: 1 ps RMS; SJ: 0.5 ps RMS
Transient simulation	30,000 UI; 400 samples/UI Rising and falling crossings
Consistency	Same linear channel $H(f)$ Delay-stripped interpolation

Jittered TX waveform → Linear channel filtering → RX zero crossings

- Compare frequency-domain prediction with direct time-domain crossing extraction.

Case 1: A Smooth Lossy Channel



Transmission line
f_c = 4.4 GHz

Low-offset gain ≈ 1

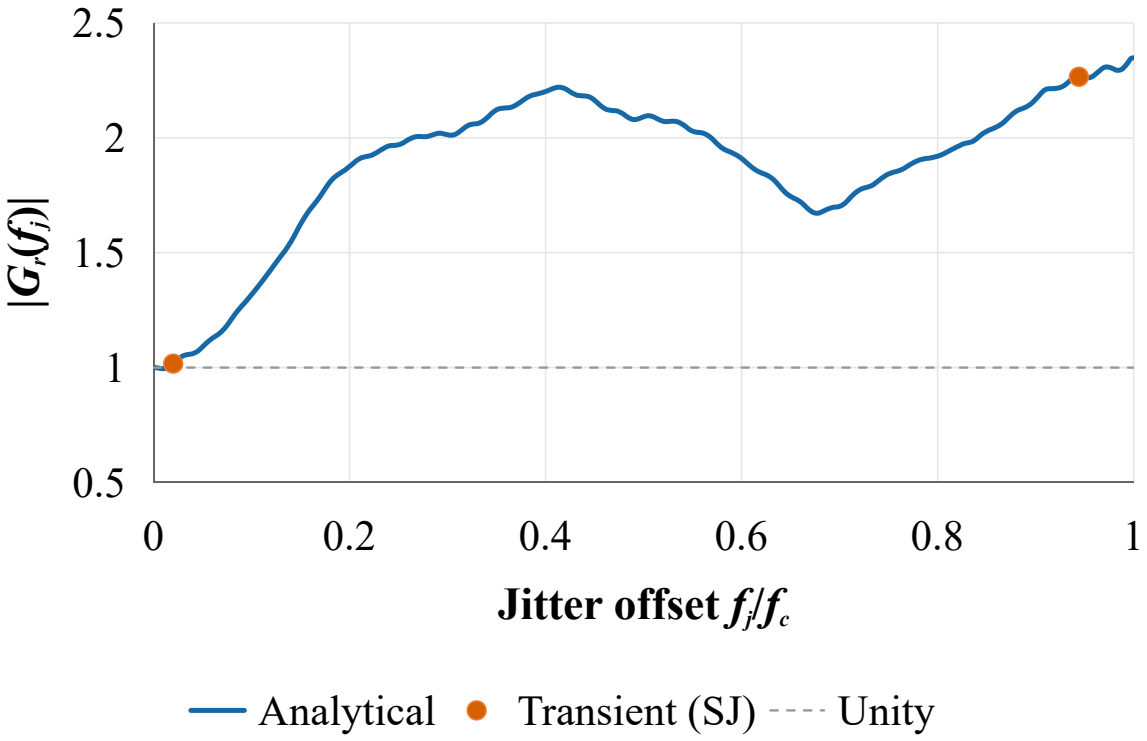
Higher offsets show larger timing gain.

Markers: fitted sinusoidal timing response.

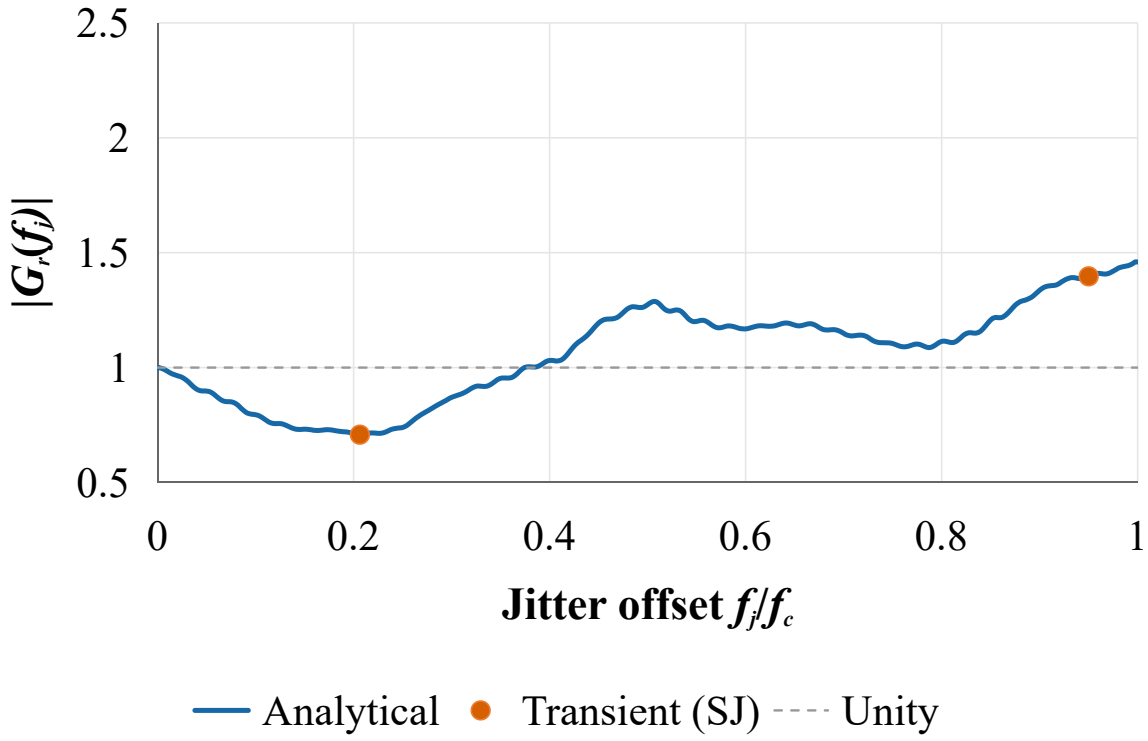
- Smooth loss provides a baseline for interpreting sideband transfer.

Case 2: Jitter Transfer in DDR channel

DDR channel f_c = 3.4 GHz



DDR channel f_c = 4.0 GHz



Amplification over most offsets

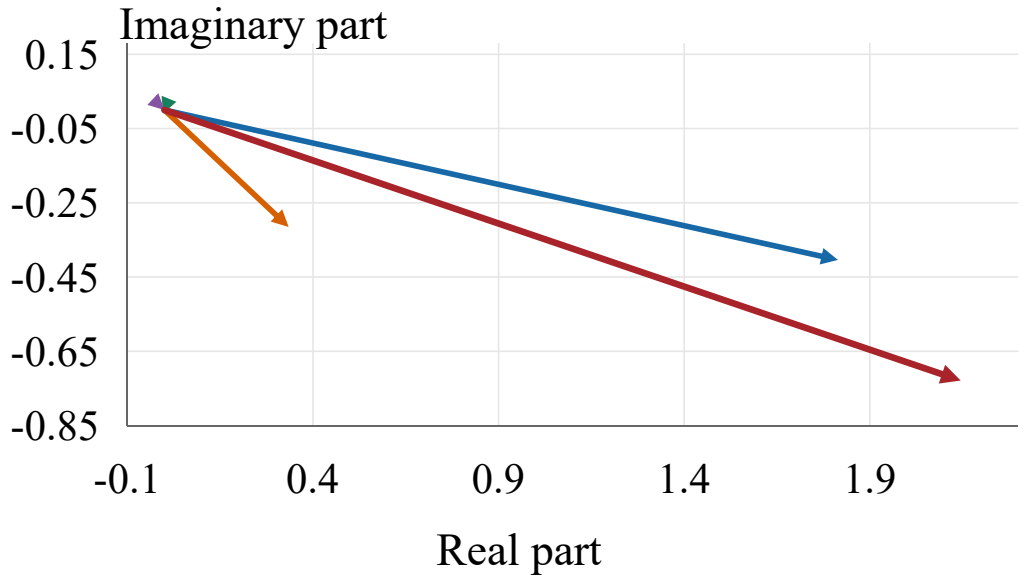
Low-offset attenuation

- Changing the clock frequency changes the entire jitter-transfer response.

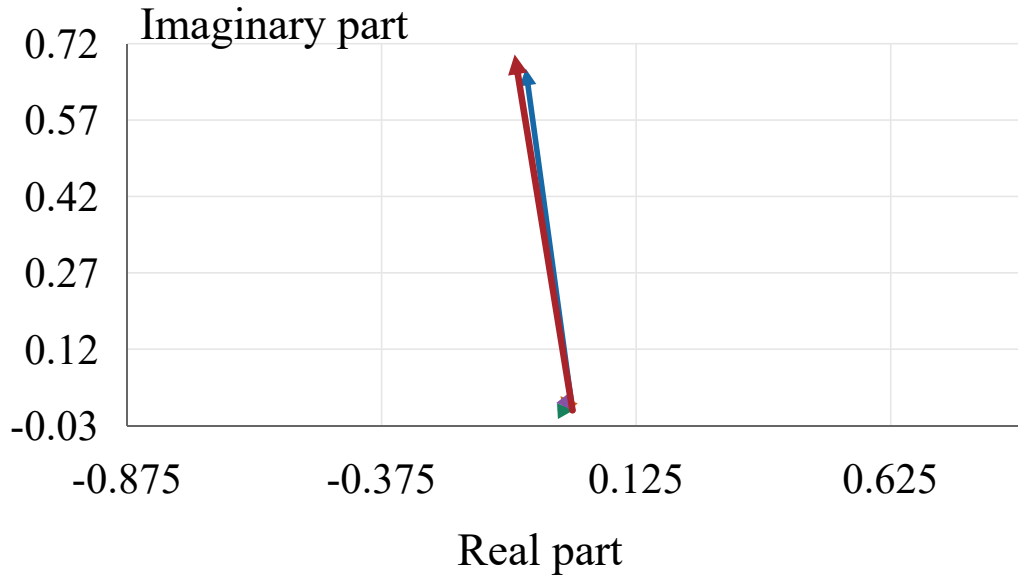
Coherent Harmonic Contributions

Q_p: contribution from the ±p harmonic pair; $G_r(f_j) = \Sigma Q_p$

f_c = 3.4 GHz; f_j = 3.21 GHz



f_c = 4 GHz; f_j = 0.8255 GHz



±1 ±3 ±5 Other harmonic contributions Total jitter-transfer response

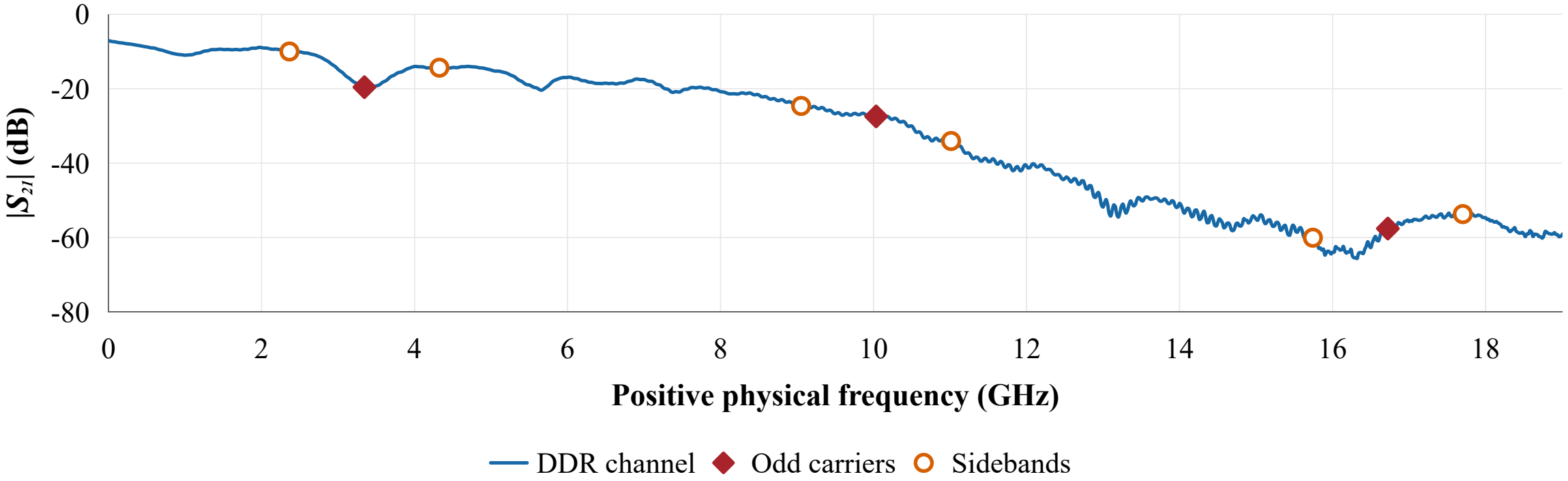
$\Sigma|Q_p|^2 = 3.665$; cross = +1.466
$|\Sigma Q_p| = 2.265$

$\Sigma|Q_p|^2 = 0.460$; cross = +0.041
$|\Sigma Q_p| = 0.708$

- Both cross terms are positive. Attenuation also reflects smaller contributions.

Carrier and Sideband Positions

DDR channel f_c = 3.3445 GHz; f_j = 0.98 GHz



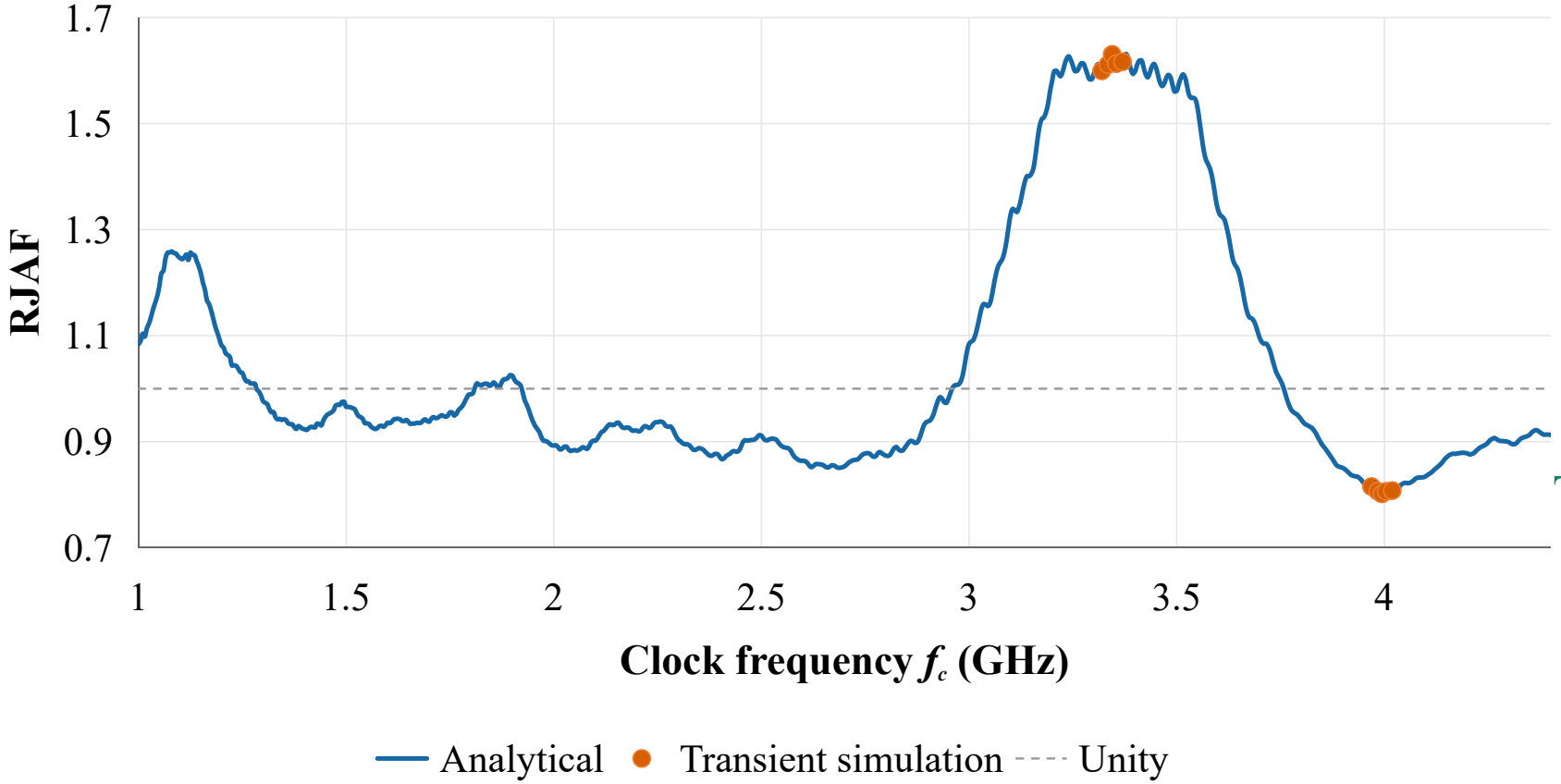
The fundamental lies in the notch region; the sidebands experience different losses.

Calculation retains negative-frequency conjugates. Phase and the actual crossing slope also matter.

- Frequency locations guide interpretation; the complex response completes it.

Clock Frequency at Fixed Jitter Bandwidth

DDR channel Clock frequency sweep: f_c = 1.0-4.4 GHz



DDR channel

B_j = 1 GHz
Target input=1 ps RMS

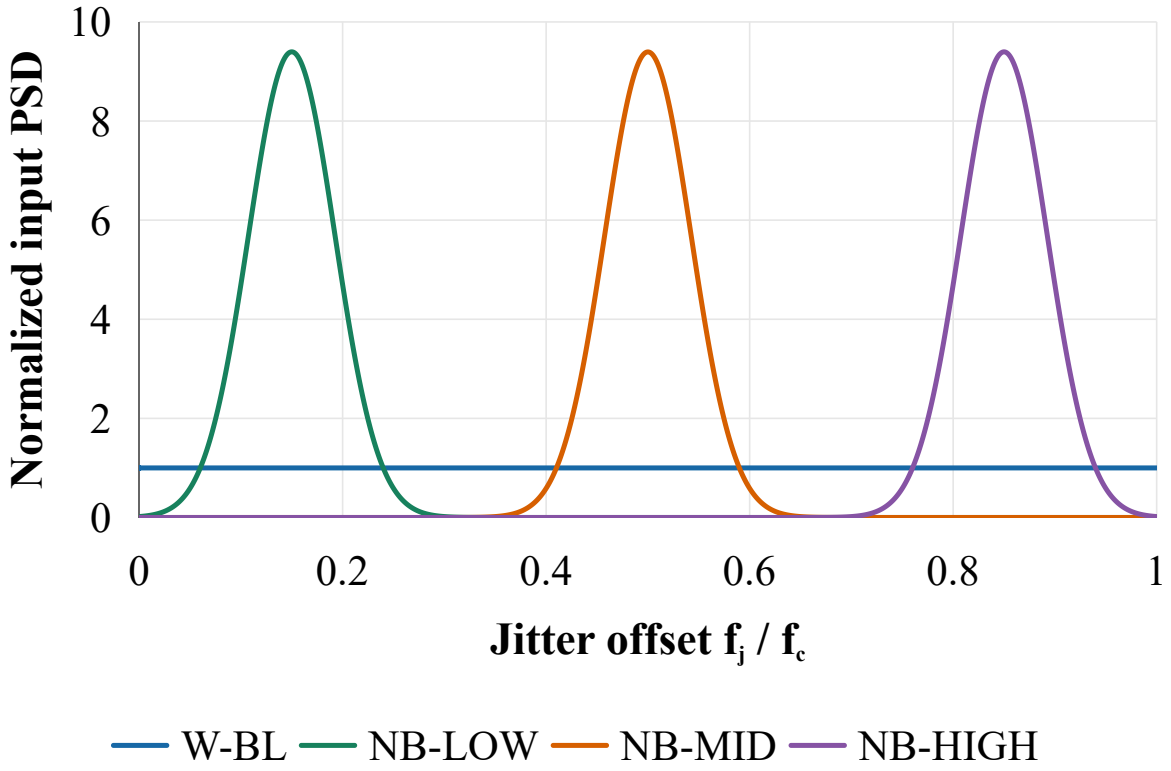
Peak region≈ 3.34 GHz / 1.63

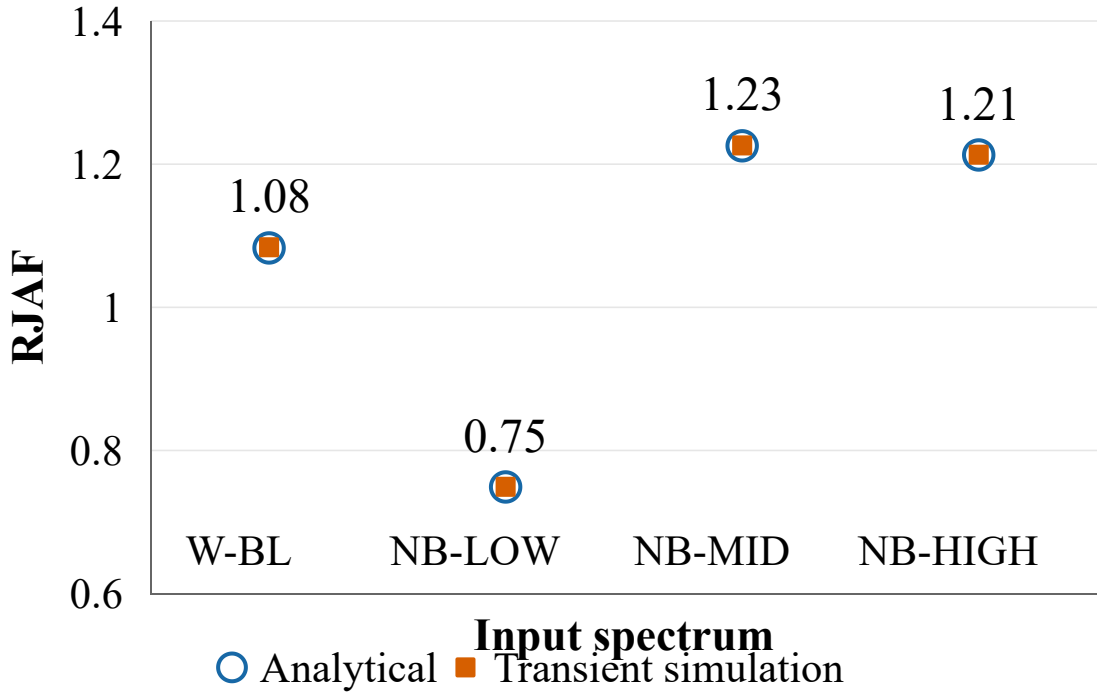
Trough region≈ 3.99 GHz / 0.80

- RJAF remains nonmonotonic when the input jitter bandwidth is fixed.

Equal RMS, Different Jitter Spectra

DDR channel f_c = 4 GHz; Target input = 1 ps RMS





Unit area in the f/f_c coordinate.

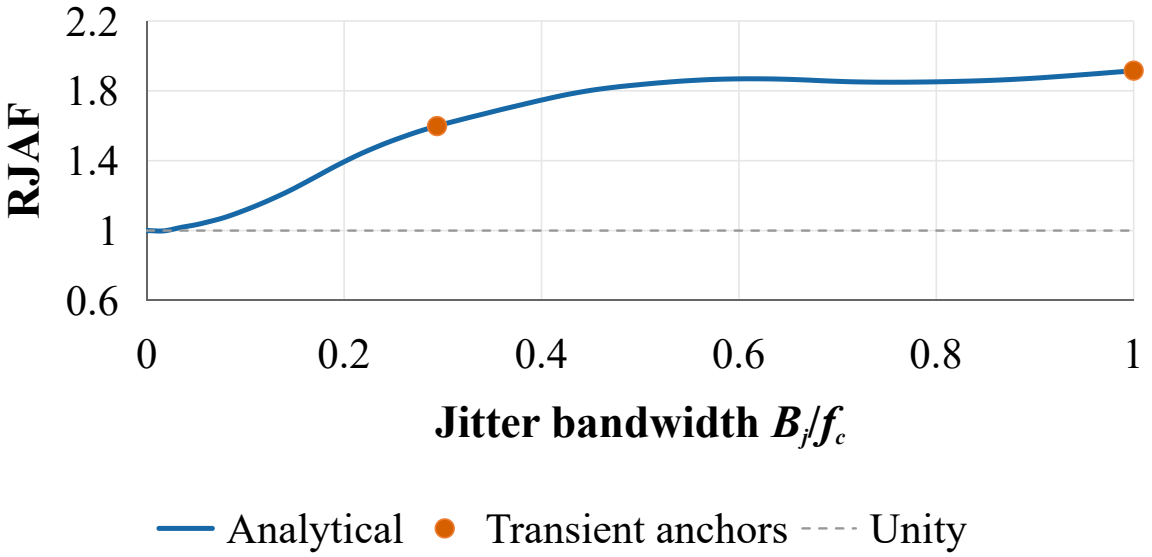
95% CI: 4 paired seeds; bars may be shorter than markers.

- Same target RMS: low-offset spectrum gives ≈ 0.75; mid-offset gives ≈ 1.23.

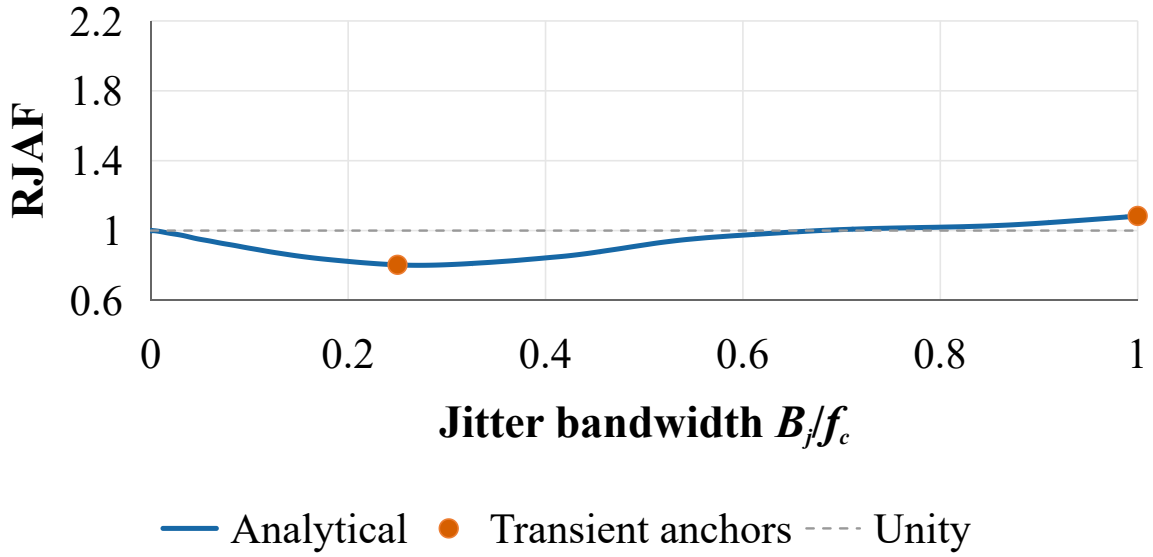
Effect of Input Jitter Bandwidth

DDR channel Band-limited white timing jitter; Target input = 1 ps RMS

f_c = 3.4 GHz



f_c = 4.0 GHz



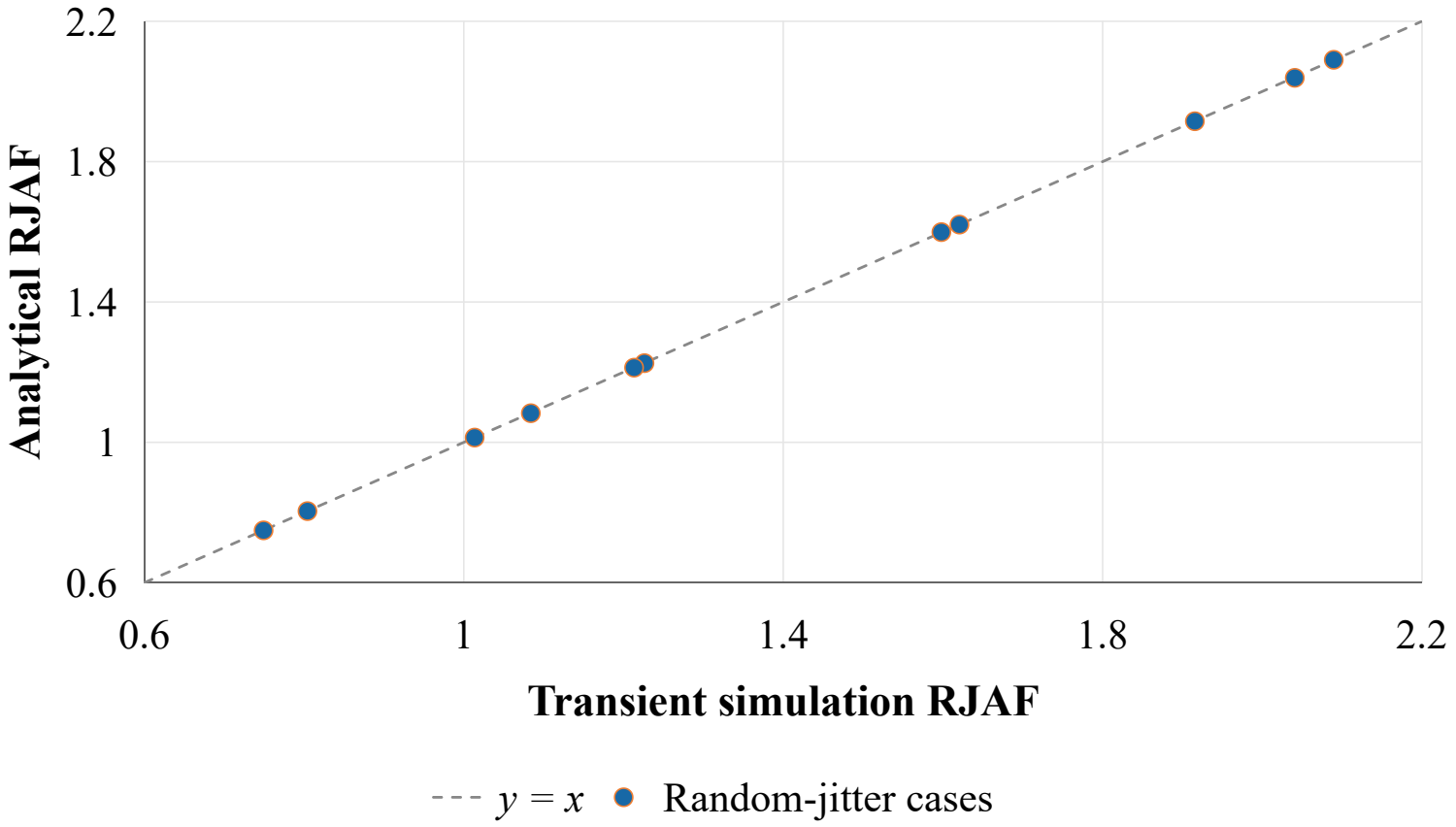
$B_j = f_c$: RJAF ≈ 1.92 (opening case)

B_j = 1 GHz: RJAF ≈ 0.804

$$C(B_j) = \int_0^{B_j} q_r(f_j)\, df_j, \qquad \mathrm{RJAF}_r^2(B_j) = \frac{C(B_j)}{B_j}$$

- At fixed total input RMS, bandwidth changes a spectral average.

Validation Against Transient Simulation



18 representative cases

11 random-jitter cases
7 sinusoidal-jitter cases

Random jitter: max difference 0.0879%

SJ fitted magnitude: 0.0123%
SJ phase: 0.00264°

3 target-theory values lie outside the paired-seed 95% CI.

- Validation supports the stated linear-channel, small-timing-disturbance model.

Implications for Channel Evaluation

01 **Complex channel response**

Include the relevant carriers, harmonic sidebands and their phase.

02 **Actual RX crossing**

Use the complete reference waveform and signed slope K_r.

03 **Actual input jitter PSD**

Weight $q_r(f_j) = |G_r(f_j)|^2$ by the input timing spectrum.

04 **Distinct frequency variables**

Separate clock frequency f_c, jitter offset f_j and bandwidth B_j.

- Channel response, source waveform and input spectrum jointly determine output TIE.

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Conclusions and Future Work

- Clock-jitter transfer depends on the coherent combination of harmonic sidebands and the actual RX crossing slope.
- RJAF depends on the input jitter spectrum. Equal input RMS can produce different output jitter.
- Clock frequency and jitter bandwidth can change whether a channel amplifies or attenuates timing jitter.

- ➢ Extend the phase-noise-based framework to RJAF calculation for PRBS data signals.
- ➢ Account for pattern-dependent ISI and crossing slopes, and distinguish random-jitter transfer from deterministic timing shifts.

Thank you